Lecture Notes

on

Strength of Material

(Th.2)

3rd Semester, Mechanical Engg.

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CHAPTER 1.0

SIMPLE STRESS AND STRAIN

1.1 - Types of Load

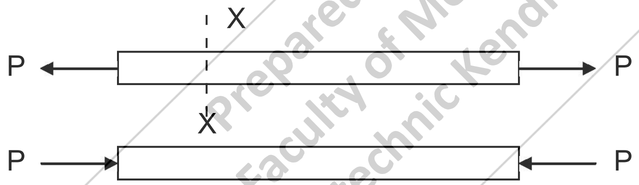
Load is an external force. Hydraulic force, steam pressure, tensile force, compressive force, shear force, spring force and different types of load. Again load may be classified as live load, dead load.

Definition

Strength of material is the study of the behaviour of structural and machine members under the action of external loads, taking into account the internal forces created and resulting deformation.

Types of load

The simplest type of load (P) is a direct pull or push, known technically as tension or compression.

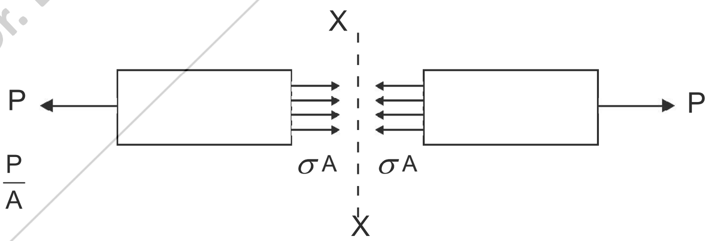


If a member is in motion the load may be caused partly by dynamic or inertia forces. For instance, the connecting Rod of a reciprocating engine, load on a fly wheel.

STRESS

Definition

The Force transmitted across any section, divided by the area of that section, is called intensity of stress or stress.



$$\sigma = \frac{P}{A}$$

Where

σ - Stress

P - Load

A - Area

σA - Internal forces of cohesion

Direct stress (Tensile / compressive)

Stresses which are normal to the plane on which they act are called direct stresses and either tensile or compressive.

Unit - N / m2

STRAIN

Stain is a measure of the measure of the deformation produced in the member by the load.

If a rod of length L is in tension and the elongation produced is L, then the direct

$$\text{strain} = \frac{\text{Elongation}}{\text{Original length}} \quad \varepsilon = \frac{X}{L}$$

Tensile strain will be positive compressive strain will be negative.

Hooke's Law

This states that strain is proportional to the stress producing it.

A material is said to be elastic if all the deformations are proportional to the load.

Principle of superposition

It states that the resultant strain will be the sum of the individual strains caused by each load acting separately.

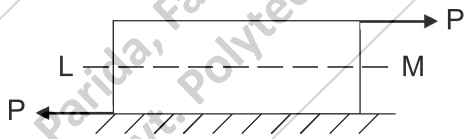
Young's Modules

Within the limits for which Hooke's law is obeyed, the ratio of the direct stress to the strain produced is called young's modules or the modules of Elasticity, i.e. E = E= $\frac{\sigma}{\varepsilon}$

For a bar of uniform cross-section A and length L this can be written as E= $\frac{PL}{AX}$ or $\frac{PL}{AE} = X$

Tangential Stress

If the applied load persists of two equal and opposite parallel forces not in the same line, then there is a tendency for one part of the body to slide over or shear from the other part across any section LM.



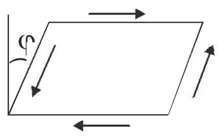
Area of gross section is parallel to load

$$\tau = \frac{P}{A}$$

Shear stress is tangential to the area over which it acts.

Every shear stress is accompanied by an equal complementary shear stress.

Shear Strain



The shear strain or slide is φ, and can be defined as the change in the right angle. It is measured in radians.

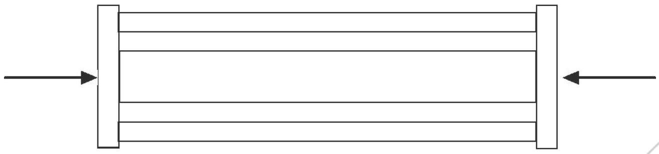
Modules of rigidity

For elastic material shear strain is proportional to the shear stress.

$$\text{Ratio } \frac{\text{Shear Stress}}{\text{Shear Strain}} = \text{Modules of rigidity}$$

$$\text{Ratio } G = \frac{\tau}{\varphi} \text{ N/mm}^2$$

1.2 Stresses in composite section



Any tensile or compressive member which consists of two or more bars or tubes in parallel, usually of different materials in called compound bars.

Analysis

A compound bar is made up of a rod of area A, and modules E1 and a tube of equal length of area A2 and modules E2. If a compressive load P is applied to the compound bar find how the load is shared. Since the road and tube are of the same initial length and must remain together then the strain in each part must be the same. The total load carried is P and let if be shared W1 and W2,

$\varepsilon_1 = \varepsilon_2$,L1=L2

compatibility equation : $\frac{W_1}{A_1 E_1} = \frac{W_1}{A_2 E_2}$

Equilibrium equation : $W_1 + W_2 = P$

Substituting, $W_2 = \frac{A_2 E_2}{A_1 E_1} \times W_1$

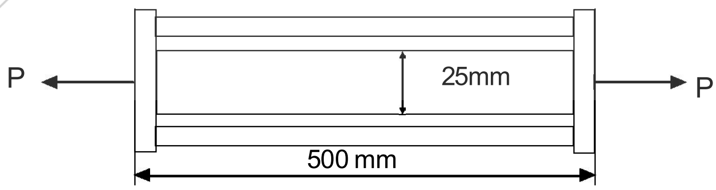
from (i) & (ii) given $W_1 \left(1 + \frac{A_2 E_2}{A_1 E_1} \right) = P$ or

$$W_1 = \frac{P A_1 E_1}{A_1 E_1 + A_2 E_2}$$

Then $W_2 = \frac{P A_2 E_2}{A_1 E_1 + A_2 E_2}$

Example

A composite bar is made up of a brass rod of 25m diameter enclosed in a steel tube, being co-axial of 40mm external diameters and 30mm internal diameter as shown below. They are securely fixed at each end. If the stress in brass and steel are not to exceed 70MPa and 120 MPa respectively find the load (P) the composite bar can safely carry.



Also find the change in length, if the composite bar is 500mm long. Take E for steel Tube as 200 GPa and brass rod as 80 GPa respectively.

Data Given

Let steel tube denoted as 1 and brass rod denoted as 2

d10= 40mm E1 = 200GPa

d1i = 30mm E2 = 80 GPa

d2 = 25mm

$\sigma 1$= 120 MPa W1 - Load carried by tube

$\sigma 1$= 70 MPa W2 - Load carried by rod.

From compatibility equation :

$$\frac{W_1}{A_1 E_1} = \frac{W_2}{A_2 E_2}$$

$$A_1 = \frac{\pi}{4}(d_{1_O}^2 - d_{1_i}^2) = \frac{\pi}{4}(40^2 - 30^2)$$

$$\Rightarrow A_1 = 500 mm^2$$

$$\text{and } A_2 = \frac{\pi}{4}\, 25^2 = 491 mm^2$$

Now putting in equation – (1)

$$\Rightarrow W_1 = W_2 \times \frac{550 \times 200}{491 \times 80}$$

$$\Rightarrow W_1 = 2.8 W_2$$

$$W_1 = \sigma_1 A_1 = 120 \times 550 = 66000 N$$

$$\text{and } W_2 = \frac{W_1}{2.8} = \frac{66000}{2.8} = 2357 N$$

From equlibrium equation

$$\Rightarrow P = W_1 + W_2$$

$$= 66000 + 2357 = 89.57 KW \text{ (Ans)}$$

Change in length

$$\delta\ell_1 = \delta\ell_2 = \frac{W_1 \ell_1}{A_1 E_1} = \frac{66000 \times 500}{550 \times 200 \times 10^3} = 0.3 mm$$

Poisson's Ratio

The ratio between lateral strain to the liner strain is a constant which is known as poisson's ratio.

The symbol is 'μ'.

Bulk Modules

When a body is subjected to three mutually perpendicular stresses of equal intensity the ratio of direct stress to the corresponding volumetric strain is known as bulk modules.

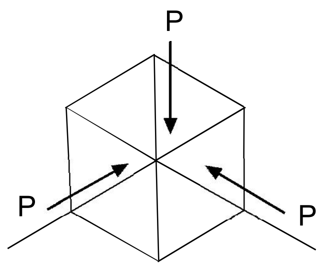


Fig. $K = \frac{-P}{\delta V / V}$

P - hydrostatic pressure

(-) - negative sign taking account of the reduction in volume.

Relation between K and E

The above figure represents a unit cube of material under the action of a uniform pressure P. It is clear that the principle stresses are -P, -P and -P and the linear strain in each direction is

$$-P/E + \mu P/E + \mu P/E = \frac{-P}{A}(1-2\mu)$$

But we know

Volumetric strain = sum of linear strain

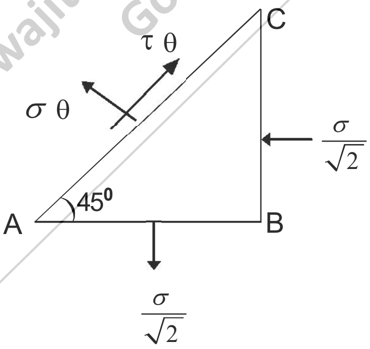
By defination $K = \dfrac{-P}{\delta V / V}$

$$\text{or } K = \frac{-P}{\frac{-3P}{E}(1-2\mu)}$$

$$\text{or } K = \frac{E}{3(1-2\mu)}$$

or **E = 3K (1-2μ)**

Relation between E and G



It is necessary first of all to establish the relation between a pure shear and pure normal stress system at a point in an elastic material.

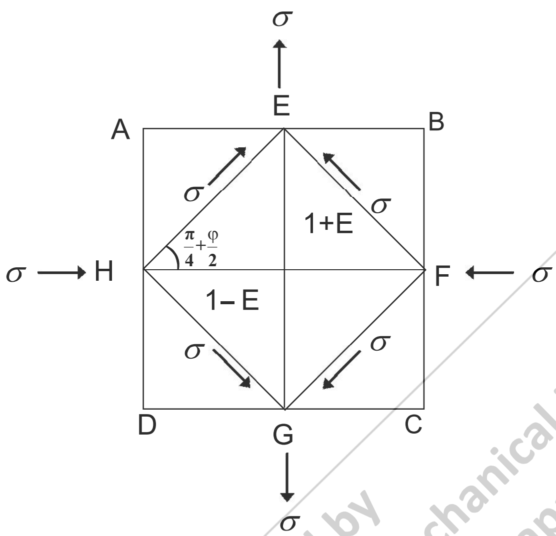
In the above figure the applied stresses are σ tensile on AB and σ compressive on BC. If the stress components on a plane AC at 45^0 to AB are σ_θ and τ_θ Then the forces acting are as shown taking the area on AC as units.

Resolving along and at right angle to AC

$$\tau_\theta = \frac{\sigma}{\sqrt{2}} \text{Sin}\, 45 + \frac{\sigma}{\sqrt{2}} \text{Cos}\, 45 = \sigma$$

$$\text{and } \sigma_\theta = \frac{\sigma}{\sqrt{2}} \text{Cos}\, 45 - \frac{\sigma}{\sqrt{2}} \text{Sin}\, 45 = 0$$

So a pure shear on planes at 45^0 to AB and BC.



This figure shows a square element ABCD, sides of unstrained length 2 units under the action of equal normal stresses, σ tension & compression. then it has been shown that the element EFGH is in pure shear of equal magnitude σ.

Liner strain in direction EG $= \frac{\sigma}{E} + \frac{\mu\sigma}{E}$

Say $\varepsilon = \frac{\mu}{E}(1+\mu)$

Liner strain in direction HF $= -\frac{\sigma}{E} - \frac{\mu\sigma}{E} = -\varepsilon$

Hence the strained lengths of EO and HO are l + ε and l - ε respectively.

The shear strain $\varphi = \frac{\sigma}{G}$

on one element EFGH and the angle EHG will increase by to $\frac{\pi}{4} + \varphi$ and angle EHO = $\frac{\pi}{4} + \frac{\varphi}{2}$

Considering the triangle tan EHO = $\frac{E0}{H0}$

$$\tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right) = \frac{1+\varepsilon}{1-\varepsilon}$$

$$\frac{1+\varepsilon}{1-\varepsilon} = \tan\frac{\tan\frac{\pi}{4} + \tan\frac{\varphi}{2}}{1 - \tan\frac{\pi}{4}.\tan\frac{\varphi}{2}}$$

$$= \frac{1+\frac{\varphi}{2}}{1-\frac{\varphi}{2}}$$

$$\varepsilon = \frac{\varphi}{2}$$

$$(1+\mu)\frac{\sigma}{\varepsilon} = \frac{\sigma}{2G}$$

then rearranging E= 2G $(1+\mu)$

by removing μ, $E = \frac{9GK}{G+3K}$

1.3 Temperature stress

Determination of temperature stress in composite bar (single core).

Temperature stresses in Composite Bar

If a compound bar made up of several materials is subjected to a change in temperature there will be tendency for the components parts to expand different amounts due to the unequal co-efficient of thermal expansion. If the parts are constrained to remain together then the actual change in length must be the same for each. This change is the resultant of the effects due to temperature and stresses condition.

Now let σ_1 = Stress in brass

ε_1 = Strain in brass

α_1 = Coefficient of liner expausion for brass

A_1 = Cross sectional area of brass bar

and σ_2, ε_2, α_2, A_2 = Corresponding values for steel.

ε = Actual strain of the composite bar per unit length.

As compressive load on the brass in equal to the tensile load on the steel, therefore

$\sigma_1 . A_1 = \sigma_2 . A_2$

strain in brass $\varepsilon_1 = \alpha_1 t - \varepsilon$

$\varepsilon_2 = \varepsilon - \alpha_2 \Delta t_2$

$\varepsilon_1 + \varepsilon_2 = \alpha_1 \Delta t_1 + \alpha_2 \Delta t_2 = \Delta t(\alpha_1 - \alpha_2)_1$

Thermal stresses in simple bar

Let L = original length of the body

Δt = Increase in temperature

α = Coefficient of liner expansion.

We know that the increase in length due to increase of temperature

$$\delta L = L \alpha \Delta t$$

$$\varepsilon = \frac{\delta L}{L} = \frac{L \alpha \Delta t}{L} = \alpha \Delta t$$

$$\text{Stress } \sigma = \varepsilon E$$

Example -1

An aluminium alloy bar fixed at its both ends is heated through 20K find the stress developed in the bar. Take modules of elasticity and coefficient of linear expansion for the bar material as 80 GPa and 24×10^{-6}/K respectively.

Data Given

Δt = 20K

E = 80GPa = 80×10^3 N/mm²

α = 24×10^{-6}/K

Solution

Then the thermal stress

$\sigma t = \alpha \Delta t E = 24 \times 10^{-6} \times 20 \times 80 \times 10^{3}$

$= 38.4\ N/mm^2 = 38.4\ mPa$

Example - 2

A flat steel bar 200mm X 20mm X 8mm is placed between two aluminium bars 200mm X 20mm X 6mm. So as to form a composite bar. All the three bars are fastened together at room temperature. Find the stresses in each bar where the temperature of the whole assembly in raised through 50^0c, Assume E_s=200GPa, E_a=80GPa, $\alpha_s = 12 \times 10^{-6}/^0c$, $\alpha_a = 24 \times 10^{-6}/^0c$

Data given

Aluminium	6mm
Steel	8mm
Aluminium	6mm

$\Delta t = 50^0c$, Es = 200GPa = 200 x 10^3 N/mm²

ε_a=80GPa = 80 x 10^3 N/mm²

$\alpha_s = 12 \times 10^{-6}/^0c$, $\alpha_a = 24 \times 10^{-6}/^0c$

Solution

As = 20 x 8 = 160 mm²

Aa = 2 x 20 x 6 = 240 mm²

$$\alpha_s = \frac{Aa}{As} \times \sigma A = \frac{240}{160} \times \sigma A = 1.5 \sigma A$$

$$\varepsilon_s = \frac{\sigma s}{\varepsilon s} = \frac{\sigma s}{200 \times 10^3}$$

$$\varepsilon_a = \frac{\sigma a}{\varepsilon a} = \frac{\sigma a}{80 \times 10^3}$$

$$\varepsilon_s + \varepsilon_a = t(\alpha_a - \alpha_s)$$

$$\frac{\sigma s}{200 \times 10^3} + \frac{\sigma a}{80 \times 10^3}$$

$$= 50(24 \times 10^{-6} - 12 \times 10^{-6})$$

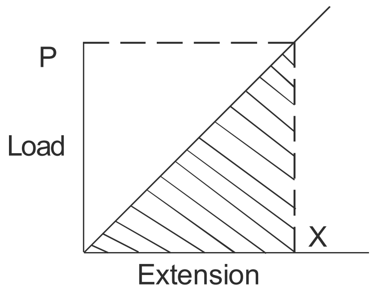
$$\text{or, } \frac{1.5 \sigma a}{200 \times 10^3} + \frac{\sigma a}{80 \times 10^3}$$

$$= 50 \times 12 \times 10^{-6}$$

$$\Rightarrow \sigma_a = 30 N/mm^2 = 30MPa$$

$$\sigma_a = 1.5 \sigma_a = 1.5 \times 30 = 45\ N/mm^2 = 45MPa$$

1.4. Strain energy resilience stress due to gradually applied load, and compact load.



Strain Energy

The strain energy (U) of the bar is defined as the work done by the load in strain it.

For a gradually applied load or static load the work done is represented by the shaded area in above figure.

$$U = \frac{1}{2} P.X$$

$$U = \frac{1}{2} \sigma A \frac{\sigma}{E} L$$

$$= \frac{1}{2E} \sigma^2 A L = \frac{1\sigma}{2E} Vol.$$

Resilience

The strain energy per unit volume usually called as resilience in simple tension or compression is $\frac{\sigma^2}{2E}$.

Proof resilience

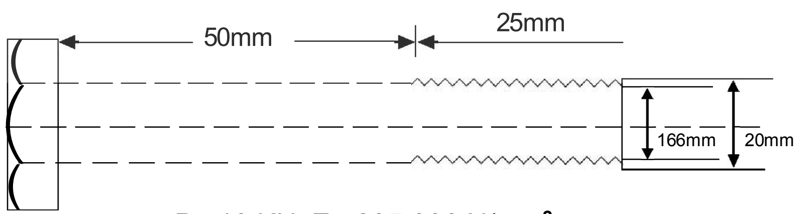
It is the value at the elastic limit or at the proof stress for non-ferrous materials.

Strain energy is always a positive quantity and being work units will be expressed as Nm (i.e. joules)

Example 1

Calculate the strain energy of the bolt as shown below under a tensile load of 10 KN. Show that the strain energy is increased for the same max stress by turning down the same of the bolt to the root diameter of the turned, E=20500 N/mm^2

Data Given



P= 10 KN, E= 205,000 N/mm^2

Solution

It is a normal practice to assume that the load is distributed events over the core.

$$A_c = \frac{\pi}{4} 16.6^2 = 217\,mm^2$$

Stress in screwed portion = $\frac{P}{A_c} = \frac{10,000}{217} = 46 N/mm^2$

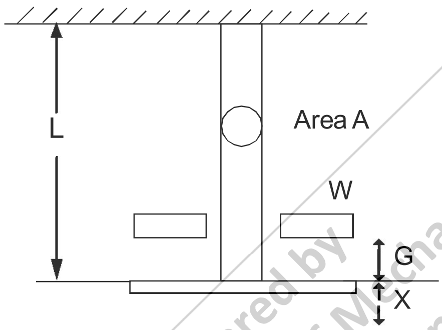
Stress in shank = $\frac{P}{A_C} = \frac{10,000}{\frac{\pi}{4} \times 20^2} = 31.8 N/mm^2$

Total strain Energy = $\frac{1}{2 \times 205000}(46^2 \times 210 \times 25 + 31.8^2 \times 314 \times 50) = 67N/mm^2$

If turned to 16.6mm

$$S.E = \frac{1}{2 \times 205000}(46^2 \times 217 \times 75) = 84N/mm$$

Impact load



Supposing a weight W falls through a height 'h' on to 'a' collar attached to one end of a uniform bar, the other end being fined. Then an extension will be caused which is greater than that due to one application of the same load gradually applied.

Let X is the maximum extension, set up and the corresponding strain is σ.

Let P be the equivalent static load which would produced the same extension X.

Then the strain energy at this instant = $E1 = \frac{1}{E}(\sigma_1 - \mu\sigma_2)$

$$\text{or } E1 = \frac{Pd}{4t_1E}(2-\mu)$$

Neglecting loss of energy at compact loss of PE of weight = Gain of strain energy.

$$w(h+x) = \frac{1}{2}Px$$

$$\text{or } w(h + \frac{PL}{AE}) = \frac{1}{2}P^2L/AE$$

Rearranging and multiplying through AE/L

$$P^2/2 - WP - WhAE/L = 0$$

Solving and discarding the negative root

$$P = W + \sqrt{W^2 + 2WGAE/L}$$

$$= W[1 + \sqrt{1 + 2hAE/WL}]$$

From which $X = \frac{PL}{AE}, \sigma = \frac{P}{A}$ can be found

When $h = 0, P = 2W$

i.e. the stress produced by a suddenly applied load is twice the static stress. Ex- Referring figure-1, let a mass of 100Kg falls 4cm on to a collar attached to a bar of 2 cm dia, 3mm long find max stress, E= 205,000N/mm²

$$\sigma = \frac{P}{A} = \frac{W}{A}[1 + \sqrt{1 + 2hAE/WL}]$$

$$= \frac{981}{100\pi}, [1 + \sqrt{1 + \frac{2 \times 40 \times \pi 100 \times 205000}{981 \times 3 \times 1000}}]$$

$$= 134N/mm^2$$

CHAPTER 2.0.

THIN CYLINDER AND SPHERICAL SHELL UNDER INTERNAL PRESSURE

2.1. Definition of hoop stress

By symmetry the three principal stresses in the shell will be the

(i) circumferential or hoop stress

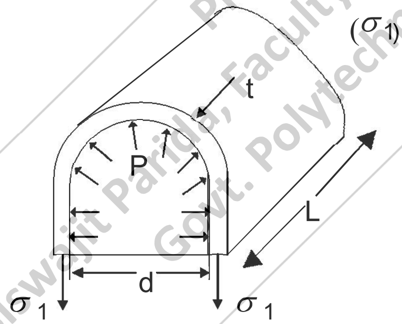
(ii) longitudinal stress

(iii) radial stress.

Thin cylinder :

If the ratio of thickness to internal diamer is less than about 1/20, then the hoop stress and longitudinal stress are constant over the thickness and the radial stress is small and can be neglected.

2.2 Hoop stress or circumferential stress derivation



Let d - internal diameter

l - length of cylinder

t - thickness

p - pressure

consider the equilibrium of a half cylinder of length L.

section through a diameteral plane, σ_1 acts on an area 2tL and the resultant vertical pressure force is found from the projected area horizontal d x L

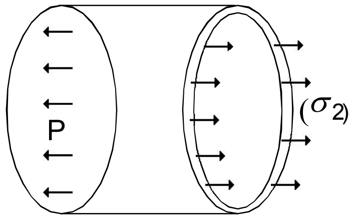
Equating forces

$\sigma_1 \times 2 \times tL = P \times d \times L$

$= \sigma_1 = \frac{PD}{2t}$

hoop stress in a tensile stress acts circumferentially on the cylinder.

Longitudinal stress σ_2 Derivation



Consider the equilibrium of a section cut by a transverse plane, σ_2 acts on an area π_2, dt (d should be the main diameter) and pacts on a projected area of $\frac{\pi}{4}d^2$ equating the forces.

Equating the forces

$$\sigma_2 \times dt = P \times \frac{\pi}{4} d^2$$

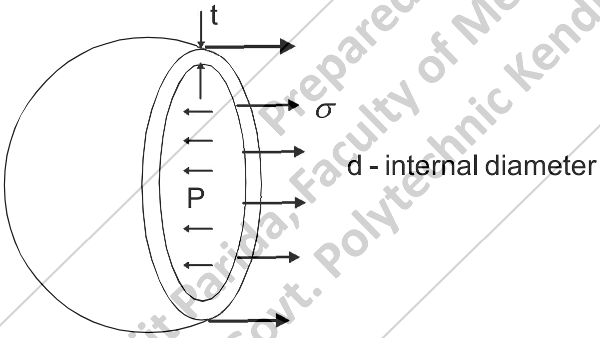
Whatever the actual shape of the end

i.e. $\sigma_2 = \dfrac{Pd}{4t}$

In case of long cylinder or tubes this stress may be neglected.

Thin spherical shell under internal pressure derivation

Again the radial stress will be neglected and the circumferential or hoop stress will be neglected and by symmetry the two principal stresses are equal, in fact the stress in any tangential direction is equal to σ.

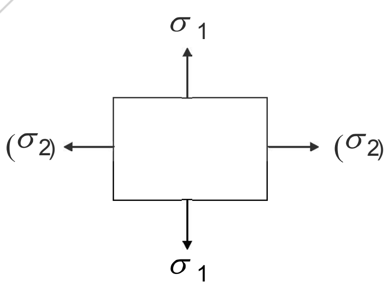


From above figure it is seen that

$$\sigma \pi dt = P \frac{\pi}{4} d^2$$

i.e. $\sigma = \dfrac{Pd}{4t}$

Volumetric strain



Hoop Strain

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \mu\sigma_2)$$

$$\text{or } \varepsilon_1 = \frac{Pd}{4t_1E}(2 - \mu)$$

Longitudinal Strain

$$\varepsilon_2 = \frac{1}{E}(\sigma_2 - \mu\sigma_1)$$

Volumetric Strain on capacity

The capacity of a cylinder $\frac{\pi}{4} d^2 L$ If the dimension is increased by δd and δL, the volumetric strain

$$= \frac{(d + \delta d)(L + \delta L) - d^2L}{d^2L}$$

$$= \frac{[d^2L + d^2\delta L + 2\delta d.dL + 2\delta d.d.\delta L + \delta d^2 L + \delta d^2 \delta L d^2 L]}{d^2L}$$

$$= (d^2\delta L + 2\delta d.\,dL) / d^2L$$

$$= 2.\delta d / d + \delta L / L$$

$$= 2 \text{ x diameteral strain} + \text{longitudinal strain}$$

$$= 2 \text{ x hoop strain} + \text{longitudinal strain}$$

Change in volume = $(2\varepsilon_1 + \varepsilon_2)$ volume

For spherical shell, volume strain = 3 x hoop strain

Change in diameter = $\varepsilon_1.d$

Change in length = $\varepsilon_2.L$

Example – 1

A gas cylinder of internal diameter 40mm is 5mm thick, if the tensile stress in the material is not to exceed 30 MPa, find the maximum pressure which can be allowed in the cylinder.

Data given

D = 40mm, t = 5m

σ_1= 30MPa = 30 N/mm2

Solution

$$\text{we know}, \sigma_1 = \frac{Pd}{2t}$$

$$\text{or, } 30 = \frac{P \text{ x } 40}{2 \text{ x } 5}$$

$$= P = 7.5 MPa$$

Example – 2

A cylindrical thin drum 80mm diameter and 4m long is made 10mm thick plates. If the drum is subjected to an internal pressure of 2.5MPa determine its changes is diameter and length. E = 200GPa.

Data given

d = 80 mm

L = 4m

T = 10mm

P = 2.5 N/mm2

E = 200 x$10^3 N/mm^2$

Solution

$$\varepsilon_1 = \frac{Pd}{4tE}(2-\mu)$$

$$\varepsilon_1 = \frac{2.5 \times 800}{4 \times 10 \times 200 \times 10^3}(2-0.25)$$

$$\delta d = \varepsilon_1 \times d = \frac{2.5 \times 800^2}{4 \times 200 \times 10^3} \times 1.75$$

$$= 0.35 mm \ \ (Ans)$$

Change in length

$$\varepsilon_2 = \frac{Pd}{2tE}(\frac{1}{2}-\mu)$$

$$\delta L = \varepsilon_2 L$$

$$= \frac{PdL}{2tE}(\frac{1}{2}-\mu)$$

$$= \frac{2.5 \times 800 \times 4 \times 10^3}{4 \times 10 \times 200 \times 10^3}(\frac{1}{2}-0.25)$$

$$= 0.5 mm (Ans)$$

Example – 3

A cylindrical vessel 2m long and 500mm dia with 10mm thick plates in subjected to an internal pressure of 3MPa, calculate the change in volume of the vessel.

E= 200GPa, μ = 0.3

Data given

$L = 2 \times 10^3$ mm

d = 500 mm

t = 10mm

P = 3MPa

$E = 200 \times 10^3 N/mm^2$

$$\varepsilon_2 = \frac{Pd}{2tE}(\frac{1}{2}-\mu)$$

$$= \frac{3 \times 500}{2 \times 10 \times 200 \times 10^3}(\frac{1}{2}-0.3)$$

$$= 0.075 \times 10^{-3}$$

$$V = \frac{\pi}{4}d^2L = \frac{\pi}{4} \times 500^2 \times 2 \times 10^3$$

$$= 392.2 \times 10^6 mm^3$$

Change in Volume

$= V (2\varepsilon_1 - \varepsilon_2)$

$= 392.7 (2x.32x10^3 + .075 \times 10^{-3})$

$= 185 \times 10^{-3} mm^3$

CHAPATER. 3.0

TWO DIMENSION STRESS SYSTEMS

3.1 Determination of normal stress, shear stress and resultant stress on oblique plane.

In many instances, however, both direct and shear stresses are brought into play, and the resultants stress across any section will be neither normal nor tangential to the plane.

If σ_r Is the resultants stress making an angle γ with the normal to the plane on which of acts.

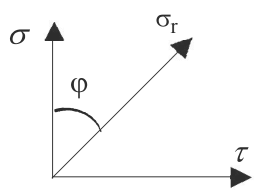


Fig 3.1

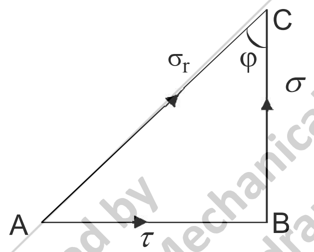


Fig 3.2

$$\varphi = \tan\frac{\tau}{\sigma}$$

$$\sigma_r = \sqrt{\sigma^2 + \tau^2}$$

Stress on oblique plane

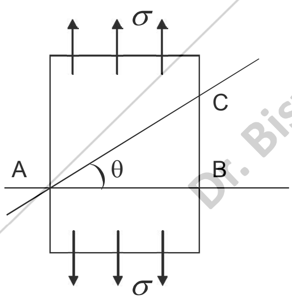


Fig 3.3

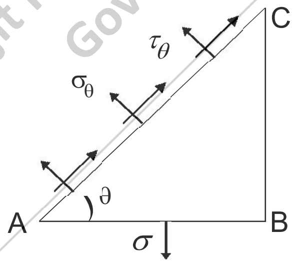


Fig 3.4

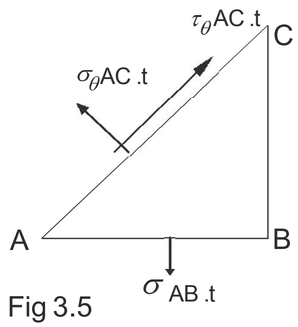


Fig 3.5

The problem is to find the stress acting on any plane AC at an angle θ to AB. This stress will not be normal to the plane, and may be resolved into two components σ_θ and τ_θ .

As per Figure 3.4 show the stresses acting on the three planes of the triangular prism ABC. There can be no stress on the plane BC, which is a longitudinal plane of the bar, the stress τ_θ must be up the plane for equilibrium.

Figure 3.5 shows the forces acting on the prism, taking a thickness t perpendicular the figure.

The equations of equilibrium resolve in the direction of σ_θ.

$$\sigma_\theta . AC.t = \sigma AB.t Cos\theta$$

$$= \sigma_\theta = \sigma \left(\frac{AB}{AC}\right) Cos\theta$$

$$= \sigma Cos^2\theta$$

Resolve in the direction τ_θ

$$\tau_\theta.AC.t=\sigma AB.t\,Sin\theta$$

$$\Rightarrow\tau_\theta=\sigma\left(\frac{AB}{AC}\right)Sin\theta$$

$$\Rightarrow\tau_\theta=\sigma Cos^2.\theta Sin\theta$$

$$\Rightarrow\tau_\theta=\frac{1}{2}\sigma Sin2\theta$$

$$\Rightarrow\sigma_r=\sqrt{(\sigma_\theta^2+\tau_\theta^2)}$$

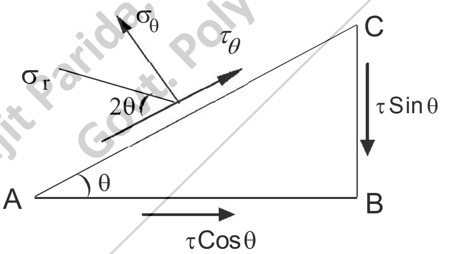
$$\Rightarrow\sigma\sqrt{Cos^4\theta+Cos^2\theta.Sin^2\theta}$$

$$\therefore\sigma_r=\sigma Cos\theta$$

It is seen that maximum shear stress is equal to one-half the applied stress and acts on planes at 45^0 to it.

Pure Shear

As the figures will always be right-angled triangles there will be no loss of generality by assuming the hypotenuse to be of unit length. By making use of these specification it will be found that the area on which the stresses act are proportional to 1 (for AC), Sin θ (for BC) and Sin θ (for AB) and future figures will show the forces acting on such an element.



Let tue τ act on a plane AB and there is an equal complementary shear stress on plane BC. The aim is to find $\sigma\theta$ & $\tau\theta$ acting on AC at[a] angle θ to AB.

Resolving in the direction of σ_θ

$$\sigma_\theta x1=(\tau Cos\theta)Sin\theta+(\tau Sin\theta).Cos\theta$$

$$=\tau Sin2\theta$$

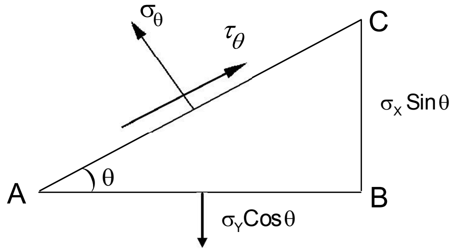
Resolving in the direction of τ_θ

$$\tau_\theta x1=(\tau Sin\theta)Sin\theta-(\tau Cos\theta).Cos\theta$$

$$=-\tau Cos2\theta(\theta\langle 45)\text{down to plane}$$

$$\sigma_r=\sqrt{\sigma^2{}_\theta+\tau^2{}_\theta}=\tau\text{ at }2\theta\text{ to }\tau_\theta$$

Pure Normal stresses on give planes



Let the known stresses be σ_x on BC and σ_Y on AB, then the forces on the element are proportional to those shown.

Resolving in the direction of σ_θ

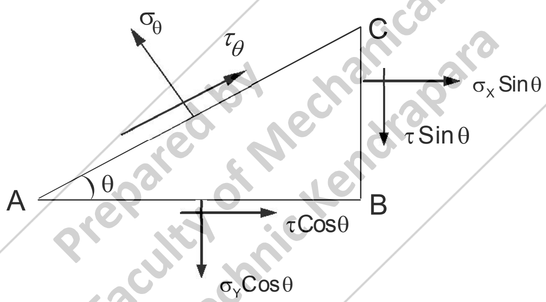
$$\therefore \sigma_\theta = \sigma_Y \, Cos^2\theta + \sigma_X \, Sin^2\theta$$

Resolving in the direction of τ_θ

$$\tau_\theta = \sigma_Y \, Cos\theta Sin\theta - \sigma_X \, Sin\theta Cos\theta$$

$$\therefore \tau_\theta = \frac{1}{2}(\sigma_Y - \sigma_X) Sin 2\theta$$

General two dimensional Stress system



Resolving in the direction of σ_θ

$$\sigma_\theta = \sigma_Y \, Cos\theta Cos\theta + \sigma_X \, Sin\theta Sin\theta + \tau Cos\theta Sin\theta + \tau Sin\theta Cos\theta$$

$$= \sigma_Y \left(\frac{1 + Cos^2\theta}{2}\right) + \sigma_X \left(\frac{1 - Cos^2\theta}{2}\right) + \tau Sin^2\theta$$

$$= \frac{1}{2}(\sigma_Y + \sigma_X) + \frac{1}{2}(\sigma_Y - \sigma_X) \tau Cos^2\theta + \tau Sin^2\theta$$

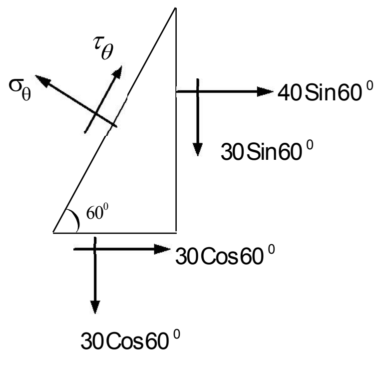
Resolving in the direction of τ_θ

$$\tau_\theta = \sigma_Y \, Cos\theta Sin\theta - \sigma_X \, Sin\theta Cos\theta - \tau Cos\theta Cos\theta + \tau Sin\theta Sin\theta$$

$$\therefore \tau_\theta = \frac{1}{2}(\sigma_Y - \sigma_X) Sin 2\theta - \tau Cos 2\theta$$

Example – 1

If the stress on two perpendicular planes through a point are 60 N/mm2 tension, 40 N/mm2 compression and 30 N/mm2 shear find the stress components and resultant stress on a plane at 60^0 to that of the tensile stresses.



Resolving

$$\sigma_\theta = 60\,Cos60^0 .\ Cos60^0 - 40\,Sin60^0 .Sin60^0 + 30\,Cos60^0\,Sin60^0 + 30\,Sin60^0\,Cos60^0$$

$$= 60\,x\frac{1}{2}\,x\frac{1}{2} - 40\,x\frac{\sqrt{3}}{2}\,x\frac{\sqrt{3}}{2} + 30\frac{1}{2}\,x\frac{\sqrt{3}}{2} + 30\,x\frac{\sqrt{3}}{2}\,x\frac{1}{2}$$

$$= 15 - 30 + 7.5\sqrt{3} + 7.5\sqrt{3}$$

$$\boxed{= \sigma_\theta = 11 N/mm^2}$$

and

$$\tau_\theta = 60\,Cos60^0 .\ Sin60^0 + 40\,Sin60^0 .Cos60^0 - 30\,Cos60^0\,Cos60^0 + 30\,Sin60^0\,Sin60^0$$

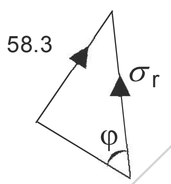
$$= 15\sqrt{3} + 10\sqrt{3} - 7.5 + 22.5$$

$$= 58.3\ N/mm^2$$

$$= \sigma_r = \sqrt{(112 + 58.32)} = 59.3 N/mm^2$$

at angle to the

$$\gamma = \tan^{-1}\frac{58.3}{11} = 80^0\,15^0$$



(20^0 to the 60 N/mm²)

Principal Planes

From equation

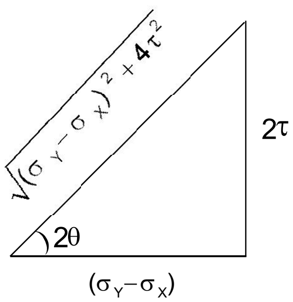
$$\tau_\theta = \frac{1}{2}(\sigma_Y - \sigma_X)Sin\,2\theta - \tau Cos2\theta$$

There are values of 0 for which τ_θ is zero and the plane on which the shear component is zero are called principal planes.

From equation above.

$$\tan 2_\theta = \frac{2\tau}{(\sigma_Y - \sigma_X)} \quad (\text{when} - \tau_\theta = 0)$$

This gives two values of 2θ differing by 180^0 and hence two values of θ differing by 90^0 i.e. the principle planes are two planes at right angles.



$$Sin2\theta = \pm\frac{2\tau}{\sqrt{(\sigma_Y - \sigma_X)^2 + 4\tau^2}}$$

$$Cos2\theta = \pm\frac{\sigma_Y - \sigma_X}{\sqrt{(\sigma_Y - \sigma_X)^2 + 4\tau^2}}$$

Principal Stresses

The stresses on the principal planes will be pure normal (tension or compression) and their values are called the principal stresses.

We know,

$$\sigma_\theta = \frac{1}{2}(\sigma_Y + \sigma_X) + \frac{1}{2}(\sigma_Y - \sigma_X) x \text{ Cos}2\theta + \tau \text{Sin}2\theta$$

Principalstresses =

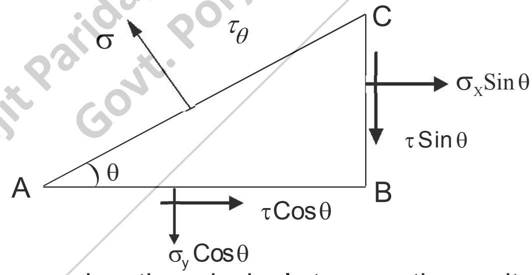
$$\frac{1}{2} x (\sigma_Y + \sigma_X) \pm \frac{\frac{1}{2}(\sigma_Y - \sigma_X)^2}{\sqrt{(\sigma_Y - \sigma_X)^2 + 4\tau^2}}$$

$$\pm \frac{\tau.2\tau}{\sqrt{(\sigma_Y - \sigma_X)^2 + 4\tau^2}}$$

$$= \frac{1}{2} = (\sigma_Y + \sigma_X) \pm \frac{\frac{1}{2}[(\sigma_Y - \sigma_X)^2 + 4\tau^2]}{\sqrt{(\sigma_Y - \sigma_X)^2 + 4\tau^2}}$$

$$= \frac{1}{2} x (\sigma_Y + \sigma_X) \pm \frac{1}{2}\sqrt{(\sigma_Y - \sigma_X)^2 + 4\tau^2}$$

Shorter method for principal stresses



Let AC be a principal plane and σ the principal stress acting on it σ_X, σ_y and τ are the known stress on planes BC and AB as before.

Resolve in the direction of σ_X

$\sigma \text{Sin}\,\theta = \sigma_X \text{Sin}\theta + \tau \text{Cos}\,\theta$

or $\sigma - \sigma_x = \tau \text{Cos}\theta$ ......(1)

Resolve in the direction of σ_y

$\sigma \text{Cos}\,\theta = \sigma_y \text{Cos}\,\theta + \tau \text{Sin}\,\theta$

or $\sigma - \sigma_y = \tau \tan\theta$ ......(2)

Multiply corresponding sides of equations (1) and (2) i.e.

$(\sigma - \sigma_x)(\sigma - \sigma_y) = \tau^2$

or $\sigma^2 - (\sigma_x + \sigma_y)\sigma + \sigma_x\sigma_y - \tau^2 = 0$

Solving

$ax^2 + bx + c = 1$

$$x = \frac{-1b \pm \sqrt{b^2 - 4ca}}{2a}$$

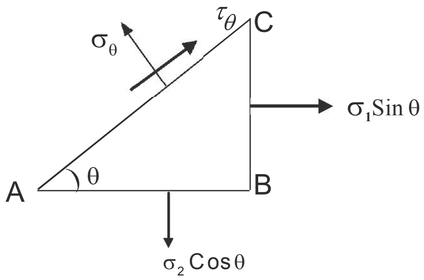
Here

$$\sigma = \frac{(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x + \sigma_y)^2 - 4\sigma_x\sigma_y + 4\tau^2}}{2}$$

or $\sigma = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$

The values of 0 for the principal planes are of course found by substitution of the principal stresses values in equation (1) & (2).

Maximum shear stress



Let AB and BC be the principal planes and σ_1 and σ_2 the principal stresses.

Then resolve

$$\tau_\theta = \sigma_2 \cos\theta . \sin\theta - \sigma_1 \sin\theta . \cos\theta$$

$$= \frac{1}{2}(\sigma_2 - \sigma_1)\sin 2\theta$$

Hence the maximum shear stress occurs when $2\theta = 90^0$ i.e. on planes at 45^0 to the principal planes and its magnitude is

$$\tau_{max} = \frac{1}{2}(\sigma_2 - \sigma_1)$$

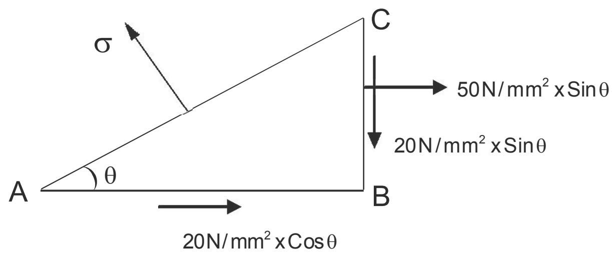
$$= \frac{1}{2}\sqrt{[(\sigma_x - \sigma_y)^2 + 4\tau^2]}$$

In words : The maximum shear stress is one-half the algebraic difference between the principal stresses.

Example – 2

At a section in abeam the tensile stress due to bending is 50 N/mm² and there is a shear stress of 20 N/mm². Determine from first principles the magnitude and direction of the principal stresses and calculate the maximum shear stress.

Solution



Resolve in the direction AB :

$$\sigma \sin\theta = 50 \sin\theta + 20 \cos\theta$$

$$\sigma - 50 = 20 \cot\theta \text{ ......(1)}$$

Resolve in the direction BC :

$$\sigma \cos\theta = 20 \sin\theta \text{......(2)}$$

$$\sigma = 20 \tan\theta$$

Multiplying corresponding sides of equations (i) and (ii)

$$\sigma(\sigma - 50) = 20^2$$

$$\sigma^2 - 50\sigma - 400 = 0$$

$$\sigma = \frac{50 \pm 10\sqrt{(25-16)}}{2}$$

$$= \frac{50 \pm 64}{2} = 57 \text{ or } -7$$

i.e. the principal stresses are 57 N/mm^2 tension, 7 N/mm^2 compression,

$$\tan\theta = \frac{\sigma}{20} = \frac{57}{20} \text{or} \frac{-7}{20}$$

Giving 0=70^0 and 160^0, being the directions of the principal planes.

Max shear stress =

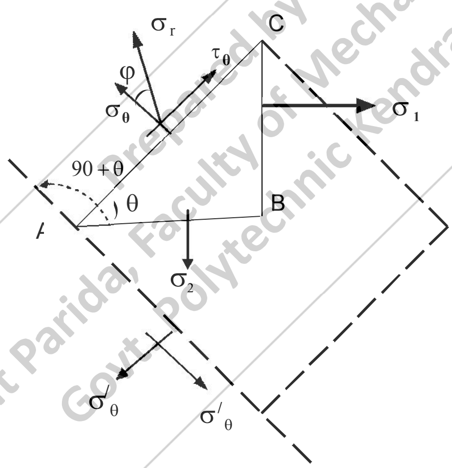
$$= \frac{1}{2}(\sigma_2 - \sigma_1)$$

$$= \frac{1}{2}[57 - (-7)]$$

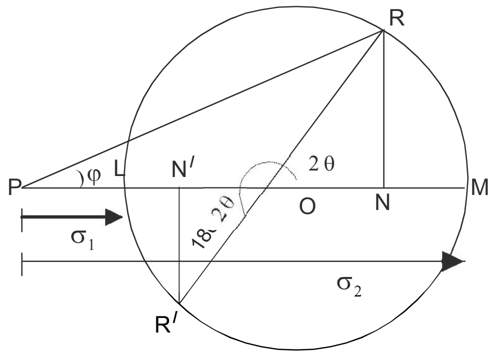
$$= 32\,N/mm^2$$

and the planes of maximum shear are at 45^0 to be principle planes i.e. 0=25^0 and 115^0. (Ans)

Maximum shear stress using Mohr's Circle



The stress circle will be developed to find the stress components on any plane AC which makes an angle θ with AB.



Construction

Mark off PL = $\sigma_1$and PM = σ_2(positive direction to the right). It is shown here for $\sigma_2 \rangle \sigma_1$, but this is not a necessary condition. On LM as diameter describes a circle center O.

Then the radius OL represents the plane of σ_1 (BC) and OM represents the plane of σ_2(AB) plane AC is obtained by rotating. AB through θ anticlockwise, and if OM on the stress circle is rotated through 2 θ in the same direction, the radius OR in obtained which will be shown to represent the plane AC.

OR could equally will be obtained by rotating OL clockwise through 180^0-2 θ , corresponding to rotating BC clockwise through 90^0- θ .

Draw RN $\perp$ r to PM

Then PN = PO + ON

$$=\frac{1}{2}(\sigma_1+\sigma_2)+\frac{1}{2}(\sigma_2-\sigma_1)\text{Cos}2\theta$$

$$=\sigma_1\frac{(1-\text{Cos}2\theta)}{2}+\sigma_2\frac{(1+\text{Cos}2\theta)}{2}$$

$$=\sigma_1\text{Sin}^2\theta+\sigma_2\text{Cos}^2\theta)=\sigma_{\boldsymbol{\theta}}$$, the normal stress component on AC

and $\text{RN}=\frac{1}{2}(\sigma_2-\sigma_1)\text{Sin}2\theta$

$=\tau_{\boldsymbol{\theta}}$, the shear stress component on AC

Also the resultant stress

$=\sigma_r=\sqrt{(\sigma^2{}_\theta+\tau^2{}_\theta)}=\text{PR}$

And its inclination to the normal of the plane is given $\varphi=\langle\text{RPN}$

σ_θ is found to be a tensile stress and τ_θ is considered positive if R is above PM,

The stresses on the plane AD, at right angles for AC, are obtained from the radius OR$^{/}$, at 180^0 to OR

i.e. $\sigma^1{}_\theta=\text{PN}^1, \tau^1{}_\theta=\text{R}^1\text{N}^1$

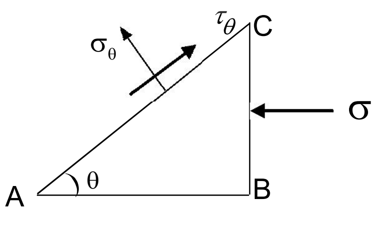
and $\tau_\theta=\tau^1{}_\theta$ but of opposite type, tending to give an anticlockwise rotation.

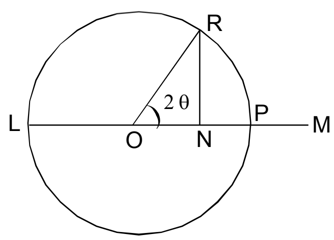
The maximum shear stress occurs when RN=OR , i.e. $\theta=45^{\mathbf{0}}$ and is equal in magnitude to $\text{OR}=\frac{1}{2}(\sigma_2-\sigma_1)$ The maximum value of φ is obtained when PR is a tangent to the stress circle.

Two particular cases which have previously been treated analytically will be dealt with by this method.

1. Pure compression

IF σ is the compressive stress the other principal stress is zero.





PL = σ numerically, measured to the left for compression, PM = 0

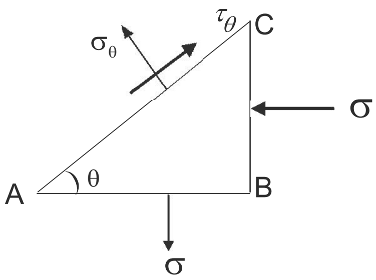
Hence, $\text{OR}=\frac{1}{2}\sigma$

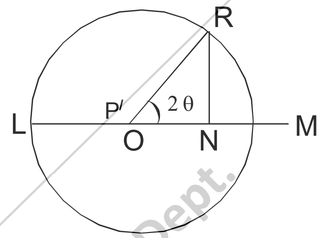
σ_θ = PN, Compressive

τ_θ = PN, Positive

Maximum shear stress = $\text{OR}=\frac{1}{2}\sigma$ occuring when $\theta=45^0$.

2. Principal stresses equal tension and compression





PM = σ to the right

PL = σ to the left

Here O coincides with P

$\sigma_\theta = PN$, is tensile for

θ between $\pm 45^0$, compressive for

θ between 45^0 and 135^0

$\tau_\theta = RN$, when $\theta = 45^0$

τ_θ reach maximum $= \sigma$, on planes when the normal stress is zero (Pure shear)

Example -3

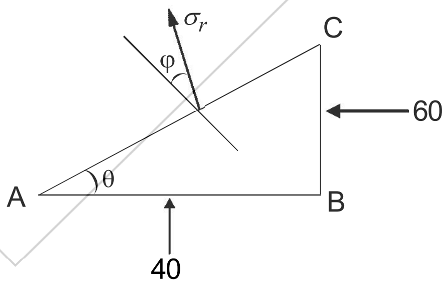
A piece of materials is subjected to two compressive stresses at right angles, their values being 40 N/mm2 and 60 N/mm2. Find the position of the plane across which the resultant stress in most inclined to the normal and determine the value of this resultant stress.

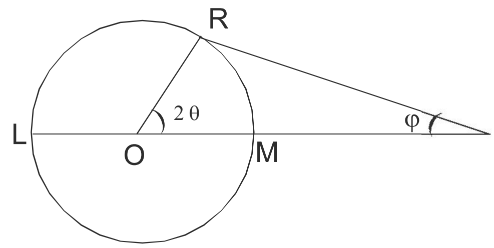
Solution

$\sigma_1 = 60 N/mm^2$ (Compressure)

$\sigma_2 = 40 N/mm^2$ (Compressure)

In the figure, the angle θ is inclined to the plane of the 40 tons N/m2 compression.





In above figure PL =60, PM=40, The maximum angle φ is obtained when PR is a tangent to the stress circle.

OR = 10, PO = 50

$$\text{Then } \varphi = \sin^{-1}\frac{1}{5} = 11^0\,30'$$

$$\sigma_r = PR = -\sqrt{(50^2 - 10^2)} = -49 N/mm^2$$

$$2\theta = 90 - \varphi$$

$$\theta = 39^0 15'$$

which gives the plane required

Example -4

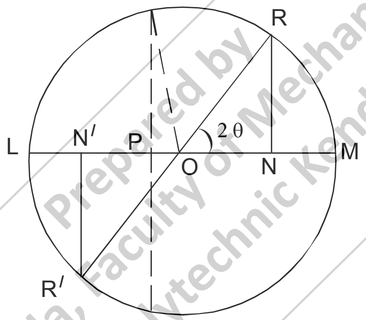
At a point in a piece of elastic material there are three mutually perpendicular planes on which the stresses are as follows : tensile stress 50 N/mm2, shear stress 40 N/mm2 on plane, compressive stress 35 N/mm2 and complementary shear stress 40 N/mm2 on the second plane, no stress on the third plane. Find (a) the principal stresses and the positions of the plane on which they act (b) the position of the planes on which there is no normal stress.

Solution

Mark off PN = 50, NR = 40

$PN' = -35,\ N'R' = -40$

Join RR', Cutting NN' at 0, Draw circle centre O, radius OR



Then $ON = \frac{1}{2} NN'$

$= 42.5$

$OR = \sqrt{42.5^2 + 40^2} = 58.4$

$PO = PN - ON = 7.5$

(a) The Principal stresses are

PM = PO + OM = 6.5 N/mm^2 (tensile)

PL = OL - OP = 50.9 N/mm^2 (compressure)

or, $2\theta = \tan\text{-}1\frac{40}{42.5} = 43^0\ 20'$

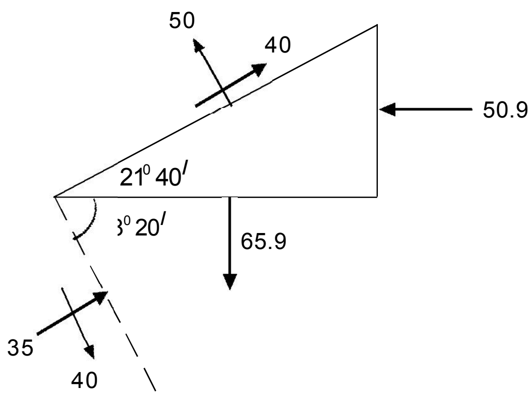
$\Rightarrow \theta = 21^0\ 40'$

(b) If there is no normal stress, then for that plane N and P coincides and

$2\theta = 180 - \text{Cos}'\ \frac{7.5}{58.4}$

$2\theta = 97^0\ 24'$

$\theta = 48^0\ 42'$ to the principal plane



CHAPTER 4.0

SHEAR FORCE & BENDING MOMENT

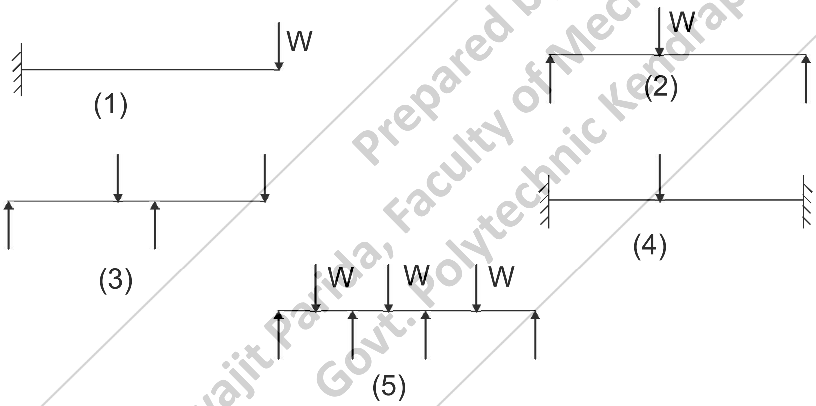
4.1 – Types of beam and load

Beam

A structural member which is acted upon by a system of external loads at right angles to its axis is known as beam.

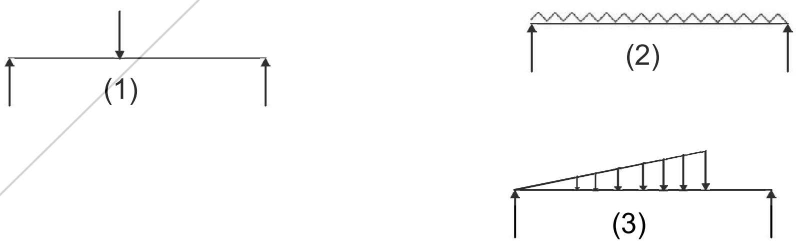
Types of Beam

1. Cantilever beam
2. Simply supported beam
3. Over hanging beam
4. Rigidity fixedor built in beams
5. Contimous beam



Types of load

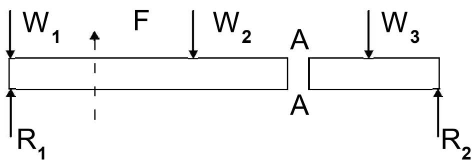
1. Concentrated or point load
2. Uniformly distributed load
3. Uniformly varying load



4.2. Concepts of share force and bending moment

Shear force

The shearing force at any section of beam represents the tendency for the portion of beam to one side of the section of slide or shear laterally relative to the other portion.

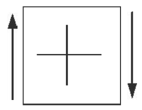


The resultant of the loads and reactions to the left of A is vertically upwards and the since the whole became is in equilibrium, the resultant of the forces to the right of AA must also be F acting down ward. F is called the shearing force.

Definition

The shearing force at any section of a beam is the algebraic sum of the lateral component of the forces on either side of the section.

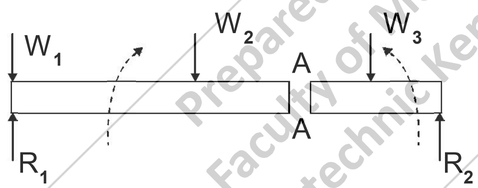
Shearing force will be considered positive when the resultant of the forces to the left is upwards or to the right in downward.



A shear force diagram is one which shows the variation of shearing force along the length of the beam.

Concepts of Bending Moment

In a small manner it can be argued that if the moment about the section AA of the forces to the left is M clockwise then the moment of the forces to the right of AA must be anticlockwise. M is called the bending moment.



Definition

The algebraic sum of the moments about the section of all the forces acting on other side of the section.

Bending moment will be considered positive when the moment on the left of section is clockwise and on the right portion anticlockwise. This is referred as sagging the beam because concave upwards. Negative B.M is termed as hogging. A BMD is one which shows the variation of bending moment along the length of the beam.

4.3 Shear force and bending moment diagram and its silent features.

i. Illustration in cantilever beam

ii. Illustration in simply supported beam

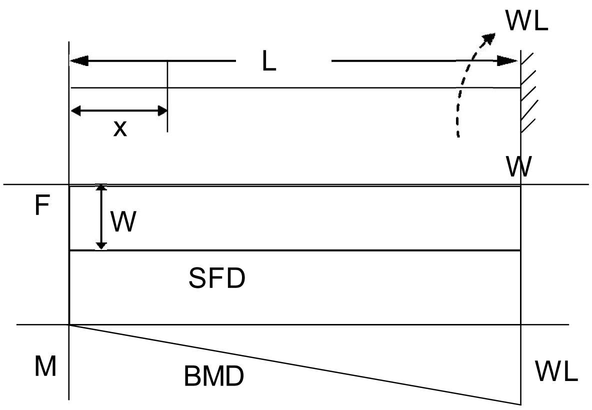
iii. Illustration in overhang beam

Carrying point load and u.d.L.

Concentrated loads

Example -1

A cantilever of length L carries a concentrated load W at its free end, draw the SF & BM diagram.



Solution

At a section a distance x from the free end, consider the forces to the left.

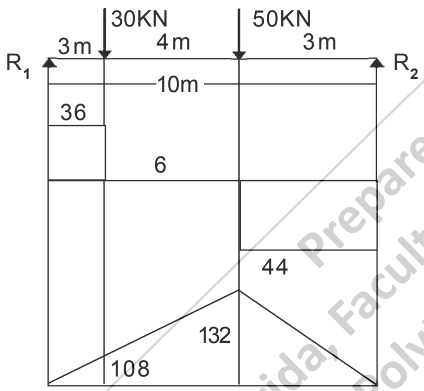
Then F = – W, and in constant along the whole beam for all values of x. Taking moments about the section given M = – Wx

A x = 0, M = 0, At –x = L, M = – WL

At end from equilibrium condition the fixing moment is WL and reactions W.

Example – 2

A beam 10m long is simply supported at its ends and carries concentrated loads of 30 KN and 50 KN at distance of 3m from each and. Draw the SF & BM diagram.



Solution

First calculate R1 and R2 at support

R1 x 10 = 30 x 7 + 50 x 3

= R1 = 36KN

and R2 = 30+50 – 36 = 44KN

Let x be the distance of the section from the left hand end.

Shearing force

O < x < 3m, F= 36KN

3 < x < 7, F= 36 -30 = 6 KN

7 < x < 10, F = 36-30-50= -44 KN.

Bending moment

0 < X , 3 M = R1 X = 36 x KNM

3 < X , 7, M = R1 X – 30 (X-3) = 6X +90 KNM

Kx < 10 , 7, M = R1 X – 30 (X-3) – 50 (X-7) = 44 X + 440 KNM

Principal values of M are

at X = 3m, m = 108 KNM

at x = 7m, M = 132 KNM

at x = 10, M = 0.

CHAPTER 4

BENDING MOMENT & SHEAR FORCE

Introduction

When any structure is loaded, stresses are induced in the various parts of the structure and in order to calculate the stresses, where the structure is supported at a number of points, the bending moments and shearing forces acting must also be calculated.

Definitions

Beam - Beam is structural member which is acted upon by a system of external loads at right angles to the axis.

Bending - Bending implies deformation of a bar produced by loads perpendicular to its axis as well as force couples acting in a plane passing through the axis of the bar.

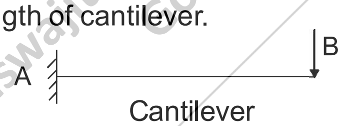
Plane bending - If the plane of loading passes through one of the principal centroidal axes of the cross section of the beam, the bending is said to be plane.

Point load - A point load or concentrated load is one which is considered to act at a point.

Distributed load - A distributed load is one which is distributed or spread in some manner over the length of the beam. If the spread is uniform, it is said to be uniformly distributed load. If the spread is not at uniform rate, it is said to be non-uniformly distributed load.

CLASSIFICATION OF BEAMS

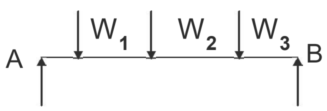
1. Cantilever – A cantilever is a beam whose one end is fixed and the other end free. Fig. 4.1 shows a cantilever with a rigidity fixed into its support and the other end B free. The length between A & B is known as the length of cantilever.



Cantilever

Fig 4.1

2. Simply supported beam – A simply supported beam is one whose ends freely rest on walls or columns or knife edges.



Simply supported beam

Fig. 4.2

3. Over hanging beam – An overhanging beam is one in which the supports are not situated at the ends i.e. one or both the ends project beyond the supports. In Fig. 4.3 C & D are two supports and both the ends A and B of the beam are overhanging beyond the supports C & D respectively.

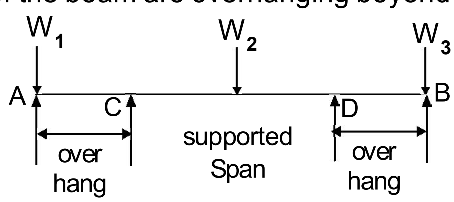
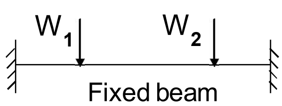


Fig. 4.3

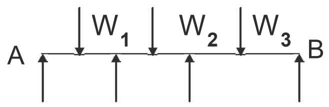
4. Fixed beam – A fixed beam is one whose both ends are rigidly fixed or built in into its supporting walls or columns.



Fixed beam

Fig. 4.4

5. Continuous beam – A continuous beam is one which has more than supports at the extreme left and right are called the end supports and all the othe the extreme, are called intermediate supports.



Continuous beam
Fig. 4.5

SHEAR FORCE

In general if we have to calculate the shear force at a section the following procedure may be adopted.

(i) Consider the left or the right part of the section.

(ii) Add the forces normal to the member on one of the parts.

If the right part of the section is chosen, a force on the right part acting downwards is positive while a force on the right part acting upwards is negative. For instance, if the shear force at a section x of a beam is required and if the right part x B be considered the forces P & θ are positive while the force R is negative. S.F. at X = P+ Q- R

Fig.4.6

If the left part of the section be chosen, a force on the left part acting upwards is positive and a force on the left part downwards is negative. For instance, if the shear force at X of a beam is required and if X A is the left part, the force Q is positive while the forces W_1 & W_2 are negative.

$\therefore$ S.F at $X = Q - W_1 - W_2$

BENDING MOMENT

To find the bending moment at a section of a beam the following procedure may be adopted.

(i) Consider the left or right part of the section.

(ii) Remove all restraints and all forces on the part selected

(iii) Now introduce each force or reacting element one at a time and find its effect at the section (i.e. find whether the moment produces a hogging or sagging effect at the section). Treat sagging moments as positive and hogging moments as negative.

Note that the moment due to every downward force is negative and moment due to every upward force is positive.

Shear force and bending moment diagrams.

A. CANTILEVER

(i) Cantilever of length L carrying a concentrated load W at the free end.

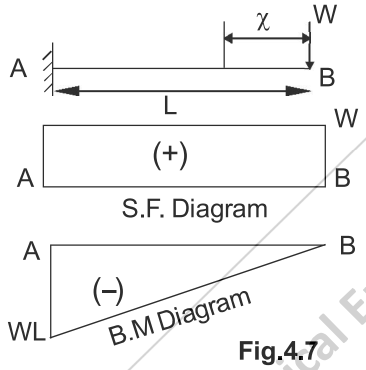


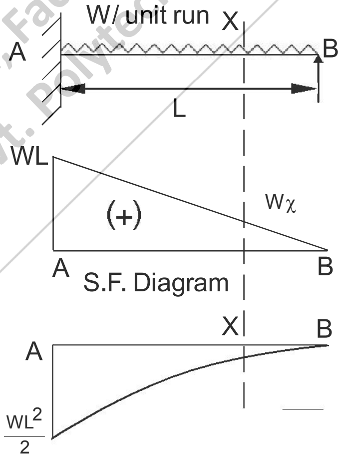
Fig.4.7

Fig. 4.7 shows a cantilever AB fixed at A and free at B and Carrying the load W at the free and B. Consider a section x at a distance of x from the free end.

S.F at X = S_χ = +W

B.M at X = M_χ = -W χ

Hence, we find that the S.F. is constant at all sections of the member between A & B. But the B.M at any section is proportional to the distance of the section from the free end.



At χ = 0 i.e. at B, B.M = 0

At χ = L i.e. at A, B.M = WL

Fig. 4.7 shows the S.F. and B.M diagrams.

(ii) Cantilever of length L carrying a uniformly distributed load of W per unit run over the whole length.

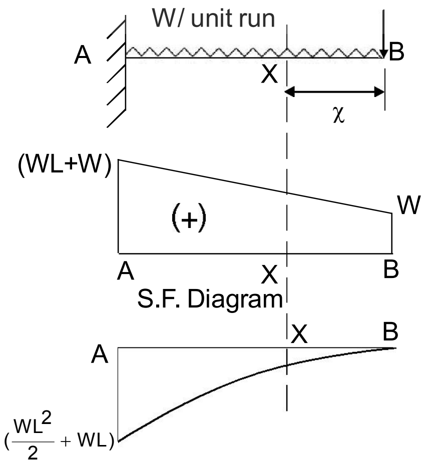


Fig 4.8 shows a cantilever AB fixed at A and free at B carrying a uniformly distributed load of W per unit run over the whole span.

Consider any section X distant χ from the end B.

S.F at X= S_χ =+W χ , B.M at X = M_χ = $-W\chi \cdot \frac{\chi}{2} = -W.\frac{\chi^2}{2}$

Hence we find that the variation of the shear force is according to a liner law while the variation of the bending moment is according to a parabolic Law.

At χ = 0, S_χ =0 M_χ =0

At χ = L, S_χ = +WL, $M_\chi = \frac{WL^2}{2}$

(iii) Cantilever of length L carrying a uniformly distributed load of W per unit run over the whole length and a concentrated load W at the free end.

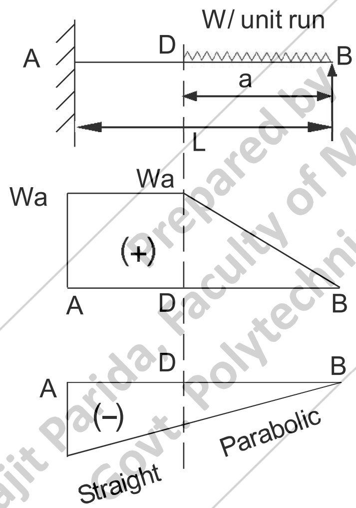


Fig.4.10

Fig. 4.10 Shows a cantilever AB fixed at A and free at B and carrying the load system mentioned above. Consider any section X distant χ from the end B. The S.F and the B.M at the section X are respectively given by

At χ = 0, S_χ =+W, $M_\chi = -(\frac{W\chi^2}{2} + WL)$

At χ = 0, S_χ =+W, M_χ =0

At χ = L, S_χ = +(WL+W), $M_\chi = +(\frac{WL^2}{2} + WL)$

S.F. varies following a liner law while B.M varies following a parabolic Law.

(iv) cantilever of length L carrying a uniformly distributed load of W per unit run for a distance a from the free end.

Fig. 4.10 shows a cantilever AB fixed at A and free at B and carrying the load system mentioned above.

Consider any section between D and B distant χ from the free end B.

S.F and B.M at the section are given by S_χ =+W χ , $M_\chi = -\frac{W\chi^2}{2}$

The above relations hold good for all values of x between χ =0 and χ =a (i.e. between B & D)

Hence for this range the S.F. varies following a linear Law while the B.M varies following a parabolic Law.

At χ = 0, S_χ =0 M_χ =0

At $\chi = a$, $S\chi = +Wa$ and $M\chi = -\frac{Wa^2}{2}$

Now consider any section between D & A, distant χ from the end B.

The S.F & B.M at this section are given by

$S\chi = +Wa$ and $M\chi = -Wa(\chi - \frac{a}{2})$

Hence between A & D, S.F. is constant at +Wa b but the B.M varies according to a linear law.

At $\chi = a$, $M\chi = -Wa\,(a - \frac{a}{2}) = -\frac{Wa^2}{2}$

At $\chi = L$, $M\chi = -Wa\;(L - \frac{a}{2})$

Problem

Fig. shows a cantilever subjected to a system of loads. Draw S.F & B.M diagrams.

Solution – At any section between D & E, distant x from E.

S.F = $S\chi$ +500kg

B.M = $M\chi = -500\chi$

At $\chi = 0$, $M\chi = 0$

At $\chi = 0.5$m, $M\chi = -500 \times 0.5 = -250$kg.m

At any section between C &D, distant χ from E,

S.F = $S\chi$ = +500+800=+1300Kg

B.M = $M\chi$ = -500x – 800 (x-0.5) = -1300x + 400

At $\chi = 0.5$, $M\chi = -1300 \times 0.5+400 = -250$Kg.m

At $\chi = 1$M, $M\chi = -1300 + 400 = -900$ Kg.m

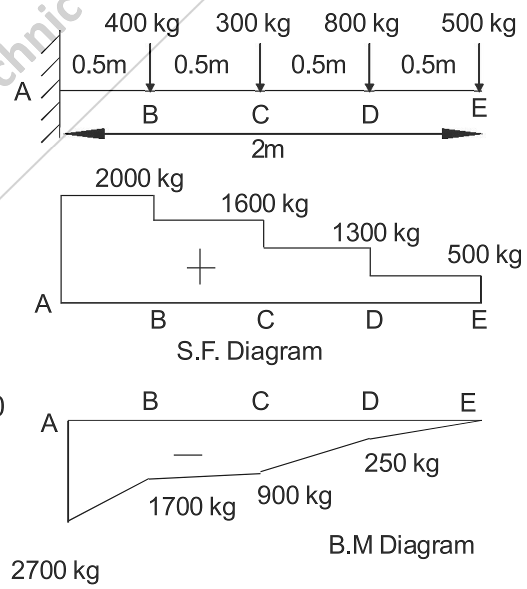


Fig. 4.11

At any section between B & E, distant x from E

S.F = $S\chi$ = +500 + 800 +300 = 1600Kg

B.M = $M\chi$ = -500x – 800(x-0.5) – 300 (x-1) Kg. M = -1600x + 700 Kg.m

At $\chi = 1$m, $M\chi = -1600 + 700 = -900$ Kg.m

At $\chi = 1.5$m, $M\chi = -1600 \times 1.5 + 700 = -1700$ Kg.m

At any section between A & B distant x from E.

S.F = $S\chi$ = +500+800+300+400 = 2000Kg

B.M = $M\chi$ = -500x -800(x-0.5)-300 (x-1) – 400 (x-1.5) = -200x + 1300Kg.m

At $\chi = 1.5$m, $M\chi = -2000 \times 1.5 + 1300 = -1700$Kg. m

At $\chi = 2$m, $M\chi = -2000 \times 2 + 1300 = -2700$ Kg.m

Beams freely supported at the two ends.

(i) Simply supported beam of span L carrying a concentrated load at mid

Fig 4.12 shows a beam AB simply supported at the ends A & B. Let the span of the beam be L and let the beam carry a concentrated load W at mid span.

Since the load is symmetrically placed on the span, reaction on the span, reaction at each support = $\frac{w}{2}$

$\therefore R_A = R_B = \frac{w}{2}$

For any section between A & C $S.F = S\chi = +\frac{w}{2}$

For any section between C & B SF = $S.F = S\chi = -\frac{w}{2}$

At the section C the S.F changes from $+\frac{w}{2}$ to $-\frac{w}{2}$

At any section between A & C distant χ from the end A, the bending moment is given by,

$M\chi = +\frac{w}{2}\chi$ (sagging moment)

At $\chi = 0$, $M\chi = 0$

At $\chi = \frac{L}{2}$, $Ma = \frac{WL}{4}$

Hence the B.M increased uniformly from zero at A to $\frac{WL}{4}$ at C.

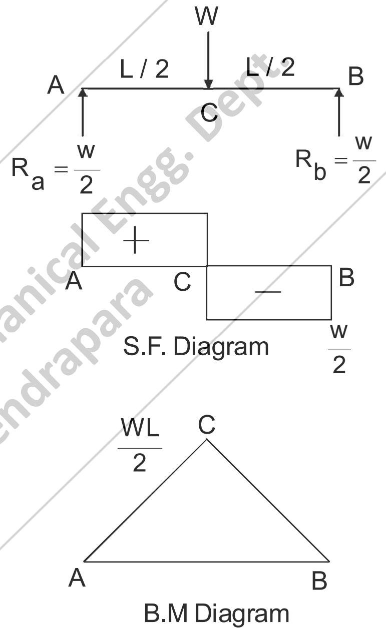


Fig. 4.12

Similarly the B.M decreases uniformly from $\frac{WL}{4}$ at C to zero at B. Maximum bending moment occurs at mid span i.e. at C where the S.F changes its sign.

(ii) Simply supported beam carrying a concentrated load placed eccentrically on the span.

Fig. 4.13 shows a simply supported beam AB of span L carrying a concentrated load W at D eccentrically on the span.

Let AD = a & DB = b

Let R_a & R_b be the vertical reactions at A & B

For equilibrium of the beam,

Taking moments of the forces on the beam about A,

we have

$R_b = Wa$

$\therefore R_b = \frac{Wa}{L}$

$\therefore R_b = W - \frac{Wa}{L} = \frac{W(L-a)}{L}$

$\therefore R_a = \frac{Wb}{L}$

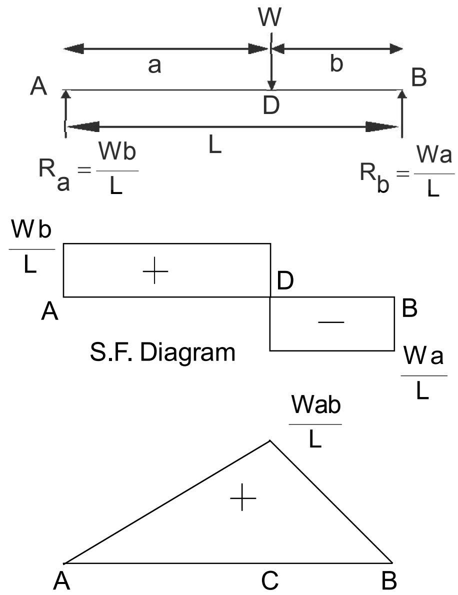


Fig. 4.13

Since a+b = L for any section between A and D

the shear force = $S\chi$ = Va = $+\frac{Wb}{L}$

For any section between D & B, the shear force = $S_\chi = -R_b + \frac{Wb}{L}$

At any section between A & D distant x from A, the bending moment is given by

$$M_\chi = +\frac{Wb}{L}\chi \text{ (sagging)}$$

At $\chi = 0, M_\chi = 0$

At $\chi = 0, M_\chi = \frac{Wab}{L}$

Hence the B.M increases uniformly from zero at the left end A to $\frac{Wab}{L}$ at D. Similarly the B.M will decrease uniformly from $\frac{Wab}{L}$ at D to zero at the right end B.

It may be observed from the S.F and B.M diagrams that the maximum B.M occurs at D where the S.F. changes its sign.

(iii) Simply supported beam carrying a number of concentrated loads.

Fig. 4.14 shows a simply supported beam AB of span 8 meters carrying concentrated loads of 4KN, 10 KN & 7 KN at distances of 1.5 meters, 4 meters & 6 meters from the left support.

S.F between C & D = + 10 – 4 = +6KN

S.F between D & E = +10 – 4 – 10 = – 4KN

S.F between E & B = +10 – 4 – 10 – 7 = –11KN

B.M at A = 0

B.M at C = +10 x 1.5 = +15KNm (Sagging)

B.M at D = +10 x 4 – 4 x 2.5 = +30 KNm (Sagging)

B.M at E = +11 x 2 = +22 KNM (Sagging)

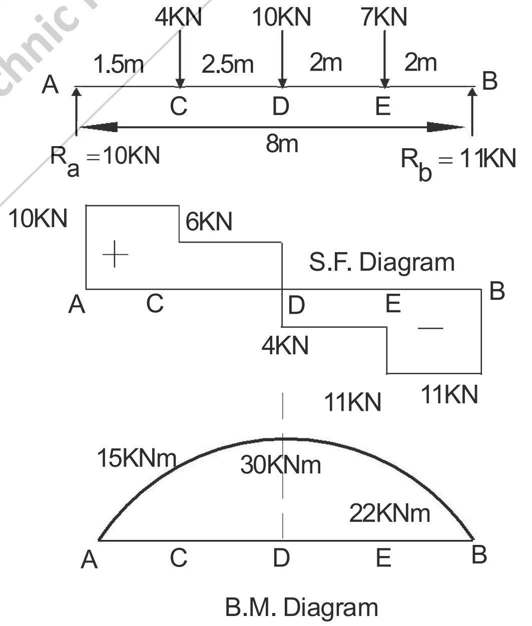


Fig. 4.14

It may be observed from the S.F & B.M diagrams that the maximum B.M occurs at D where the S.F changes its sign.

(iv) Simply supported beam carrying a uniformly distributed load of W per unit run over the whole span.

Fig. 4.15 shows a simply supported beam AB of span L carrying a uniformly distributed load W per unit run over the whole span. Let Ra & Rb be the vertical reactions at the supports A & B respectively.

Since the loading is symmetrical on the span, each vertical reaction equals half the total load on the span.

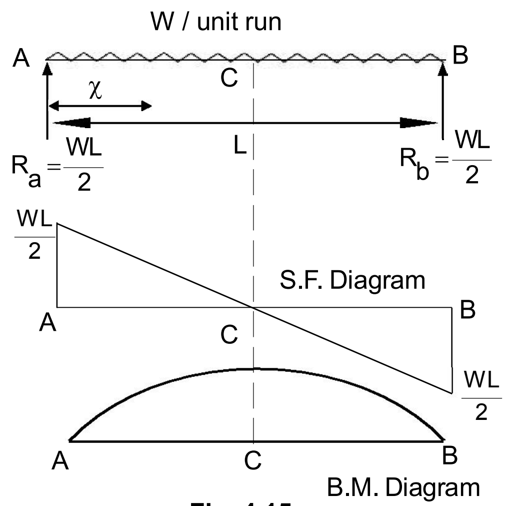


Fig. 4.15

$\therefore R_a = R_b = \frac{WL}{2}$

Consider any section X distant χ from the left end A.

S.F & B.M at the section X are given by

$$S\chi = +R_a - W\chi = +\frac{WL}{2} - W\chi$$

$$M\chi = R_a\chi - \frac{W\chi^2}{2} = \frac{WL}{2}\chi - \frac{W\chi^2}{2}$$

$$\therefore M\chi = +\frac{W}{2}\chi(L-\chi)$$

$$At\ \chi = 0, S\chi = +\frac{WL}{2}, M\chi = 0$$

$$At\ \chi = L, S\chi = +\frac{WL}{2} - WL = -\frac{WL}{2}, M\chi = 0$$

$$At\ \chi = \frac{L}{2}, S\chi = +\frac{WL}{2} - \frac{WL}{2} = 0\ \&\ M\chi = +\frac{WL}{2}.\frac{L}{2}(L-\frac{L}{2}) = +\frac{WL^2}{8}$$

The S.F diagram is a straight line. The S.F uniformly changes from $+\frac{WL}{2}$ At A to $-\frac{WL}{2}$ At B & obviously that S.F at Mid span is zero.

The B.M diagram is a parabola. The B.M increases according to a parabolic law from zero at A to $+\frac{WL^2}{2}$ at the mid span C and from this value the B.M decreases to zero at B following the parabolic law.

(v) Beam with overhanging at one end and carrying a uniformly distributed load over the whole length.

Fig. 4.16 shows a simply supported beam ABC with supports at A & B, 6 meters apart with on over hang BC 2 meters long.

Let R_a & R_b be the vertical reactions at A & B. For the equilibrium of the beam, taking moments about A,

we have Ra x 6 = 1.5 x 8 x 4

$\therefore$ Rb = 8 tones

$\therefore$ Ra = 1.5 x 8 -8 = 4 tones

S.F at the left end = +4t

S.F just on the left hand side of B = +4-1.5 x 6 = -5t

S.F. just on the right hand side of B = +1.5 x 2 = 3t

S.F at C = 0

Let S.F be zero at χ meters from A,

equating the S.F to zero,

we get S χ = 4-1.5 χ =0 $\therefore \chi = \frac{8}{3} = 2.67m$

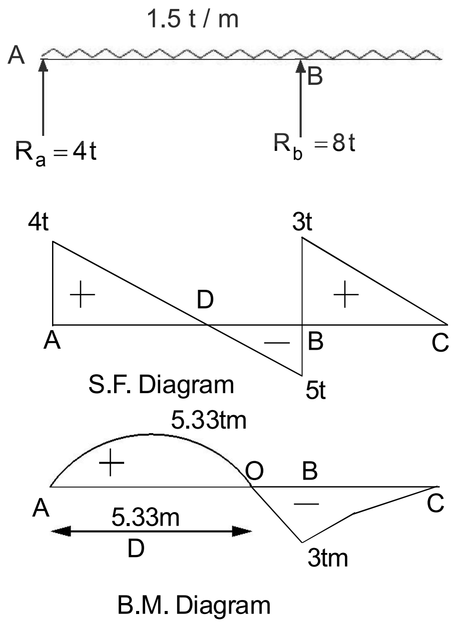


Fig. 4.16

B.M at A = 0, At any section in AB distant x from A the B.M is given by

$$M\chi = 4\chi - 1.5\frac{X^2}{2}$$

Hence the B.M diagram is parabolic

$$B.M\ at\ \chi=\frac{8}{3}MB\ \ M_{max}=4\,x\,\frac{8}{3}-\frac{1.5}{2}\left(\frac{8}{3}\right)^2=\frac{16}{3}+5.33tm$$

$$B.M\ at\ \chi=6m\ \text{i.e. at}\ B=4\,x\,6-\frac{1.5}{2}x6^2=-3tm$$

Section at which the B.M is Zero

Since at $\chi=\frac{8}{3}$ the B.M is +5.33 tm & at x = 6m the B.M is -3tm there must be a section where the B.M is zero. This section can be determined by equating the general expression for B.M to zero. i.e. by the equation

$$4\chi-1.5\frac{\chi^2}{2}=0$$

$$\therefore\chi=(4-0.75\chi)=0$$

$$\therefore\chi=0\ \&\ \therefore\chi=\frac{16}{3}=5.33m$$

Let the B.M be zero at O, AO = $\frac{16}{3}$m

The point O where the B.M is zero called the point of contra flexure or point of inflexion.

For all sections from A to O the B.M is of the sagging type while for all sections between O & C the B.M is of the hogging type.

(vi) A beam of length (L+2a) has supports L apart with an overhang a on each side. The beam carries a concentrated load W at each end. Draw S.F & B.M diagram.

Let DABC be the beam of length (L+2a). Let the supports be at A & B,

so that DA= BC =a

$\therefore$ AB = L

Each vertical reaction = W

$\therefore R_a = R_b =$ W

S.F. at any section between D & A = -W

S.F. at any section between B & C = +W

S.F. at any section between A & B = O

B.M at D = O B.M at A = -Wa

At any section in AB distant x from D the B.M is given by

Mx = -Wx + W(x-a) = -Wa

B.M at B = -Wa B.M at C = O

The B.M throughout the length is of the hogging type.

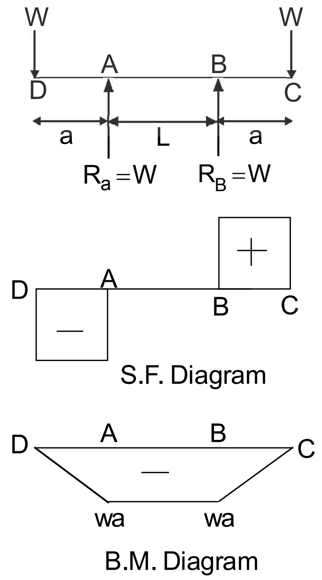


Fig. 4.17

CHAPTER 5

THEORY OF SIMPLE BENDING

When a beam is loaded it is bent and subjected to bending moments. Consequently, longitudinal or bending stresses are induced in cross section.

Assumptions in 'Theory of bending'

1. The material of the beam is perfectly homogenous

2. The stress induced is proportional to the strain & the stress should not exceed the elastic limit.

3. The value of modules of elasticity (E) is same, for the fibres of the beam under compression or tension.

4. The transverse section of the beam, which is plane before bending, remains plane after bending.

5. There is no resultant pull or push on the cross section of the beam

6. The loads are applied in the plane of bending.

7. The transverse section of the beam is symmetrical about a line passing through the centre of gravity in the plane of bending.

8. The radius of curvature of the beam before bending is very large in comparison to the transverse dimensions.

As a result of a bending moment or couple, a length of beam will take up a curved shape and a very short length may be treated as a part of the arc of circle. It follows that at the outor radii the material will be in tension and at the inner radii in compression and at some radius there will be no stress. This layer of the material is the neutral layer or neutral axis.

Fig 5.1 shows a longitudinal section of a beam, the neutral layer (axis) N.A. being bent to form an arc of a circle of radius R. The neutral layer is then, before bending, the length pq, which after bending becomes $p'q'$.

Consider some layer rs at a distance Y from pq which after bending becomes $r's'$. Let $p'q'$ subtend an angle α at the centre of curvature.

$$\therefore p'q' = R\,\alpha \text{ and } r's' = (R - y)\alpha$$

Initially the parallel layers would have equal lengths, so that Pq = rs and since there is no stress at the neutral layer, then there is no strain.

$$\therefore p'q' = pq$$

Now the strain in $rs = \dfrac{rs - r's'}{rs}$ but $rs = pq = p'q'$

$$\therefore \text{Strain} = \frac{p'q' - r's'}{rs}$$

But $p'q' = R\,\alpha$ and $r's' = (R - Y)\alpha$

$$\text{Strain } \frac{R\alpha - (R - Y)\alpha}{R\alpha} = \frac{Y}{R}$$

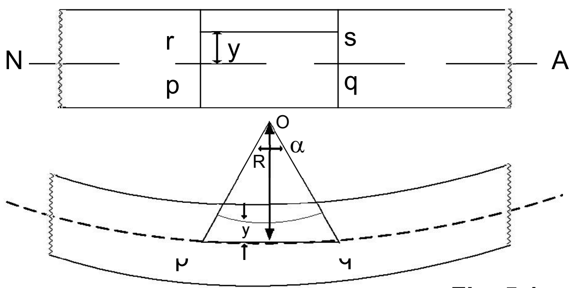


Fig. 5.1

Now if the stress in rs = σ & young's modulus = E

then strain $\frac{\sigma}{E}=\frac{Y}{R}$ or $\frac{\sigma}{Y}=\frac{E}{R}$ . . . (5.1)

If a transverse section of the beam is now considered (Fig. 5.2) let a strip of area δ a, lie at a distance Y from the neutral axis.

Then, the normal force on this area $(\delta a)=\frac{E}{R}y\,\delta a$

Now the moment of this force about the neutral axis is $=\frac{E}{R}y\delta a x y$ or $\frac{E}{R}y^2\,\delta a$

This is the resisting moment of the material caused by the stress produced and the total resisting moment is $=\Sigma\frac{E}{R}y^2\,\delta a$ or $\frac{E}{R}\Sigma y^2\,\delta a$

And $=\Sigma y^2\,\delta a$ B the second moment of area about the neutral axis, I_{NA}.

$\therefore$ Resisting moment $M\frac{E}{R}xI$

But since the resisting moment balances the applied bending moment,

$\therefore M\frac{E}{R}xI$ or $\frac{M}{I}=\frac{E}{R}$

But $\frac{E}{R}=\frac{\sigma}{Y}\therefore\frac{M}{I}=\frac{\sigma}{Y}=\frac{E}{R}$ . . .(5.2)

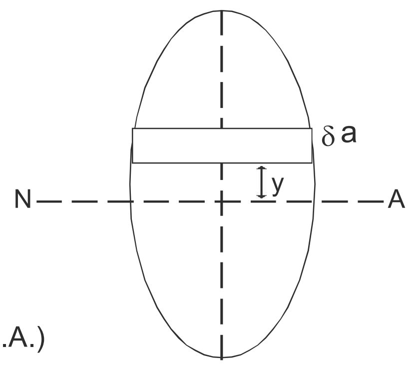


Fig. 5.2

Where,

M = moment of resistance

I = Moment of inertia of the section about neutral axis (N.A.)

E = Yong's modulus of elasticity

R = Radius of Curvature of N.A

σ = Bending stress

The above equation is known as the 'Bending equation'.

<u>Position of Neutral Axis</u>

Consider the cross-section of a beam (Fig. 5.2), there will be no resultant force on the section for condition of equilibrium.

The force acting on a small area δa at a distance 'y' from the neutral axis is given by

$SF=\sigma.\delta a=\frac{E}{R}Y.\delta a$

Or the total force normal to the section,

$$F = \frac{E}{R}\sum Y.\delta a$$

$\therefore$ For zero resultant force, $\sum Y.\delta a = 0$

Now $\sum Y.\delta a$ is the moment of the sectional area about the neutral axis and since this moment is zero, the axis must pass through the centre of area.

Hence the neutral axis or neutral layer, passes through the centre of area.

Section Modules

Referring to the bending equation, $\frac{M}{I} = \frac{\sigma}{Y}, \sigma = \frac{MY}{I}$

$$\text{or } \sigma = \frac{M}{Z} \text{ where } Z = \text{section modulus} = \frac{I}{Y}$$

The section modulus is usually quoted for all standard sections and practically is of greater use. The strength of the beam section depends mainly on the section modulus.

The section modulii of rectangular and circular sections are calculated below.

(i) Rectangular section

Fig. 5.3 shows a rectangular section of width b & depth d.

Let the horizontal centroidal axis be neutral axis.

$$\text{Section modulus } Z = \frac{\text{Moment of inertia about the neutral axis}}{\text{Distance of the most distant point of the section from the neutral axis.}}$$

$$= \frac{I}{Y_{max}}$$

But $I = \frac{bd3}{l2}$ and $Y_{max} = \frac{d}{2}$

$$\therefore Z = \frac{\frac{bd3}{l2}}{\frac{d}{2}} = \frac{bd^2}{6}$$

Moment of resistance, $M = \sigma Z = \sigma \text{ x } \frac{1}{6} bd^2$ ...(5.4)

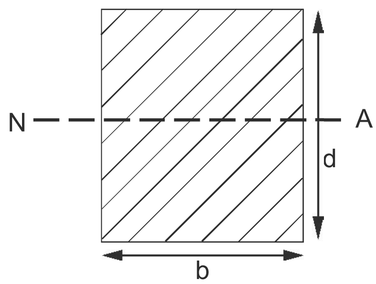


Fig. 5.3

(ii) Hollow rectangular section

Refer to Fig. 5.4.

Moment of inertia of the section about the neutral axis.

$$I = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{1}{2}(BD^3 - bd^3), \; Y_{max} = \frac{D}{2}$$

$$\therefore \text{Section modulus} = Z = \frac{I}{Y_{max}}$$

$$= \frac{(BD^3 - bd^3)/12}{D/2} = \left[\frac{(BD^3 - bd^3)}{6D}\right]$$

Moment of resistance, $M = \sigma Z = \sigma \text{ x} \left[\frac{(BD^3 - bd^3)}{6D}\right]$

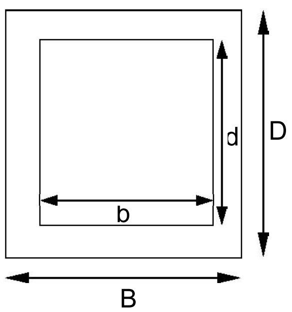


Fig. 5.4

(iii) Circular section

Refer to Fig 5.5

Moment of inertia of the section about the neutral axis.

$$I = \frac{\pi d^4}{64},\ Y_{max} = \frac{d}{2}$$

$$\therefore \text{Section modulus} = Z = \frac{I}{Y_{max}}$$

$$= \frac{\pi d^4 / 64}{d/2} = \frac{\pi d^3}{32} \quad ...(5.6)$$

$$\text{Moment of resistance, } M = \sigma Z = \sigma \times \frac{\pi d^3}{32}$$

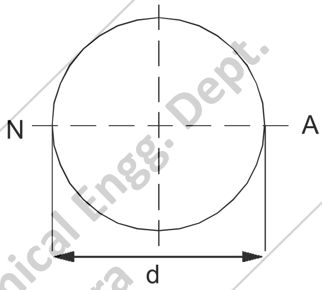


Fig. 5.5

(iv) Hollow circular section

Refer to Fig 5.6

Moment of inertia of the section about the neutral axis.

$$I = \frac{\pi}{64}(D^4 - d^4),\ Y_{max} = \frac{D}{2}$$

$$\therefore \text{Section modulus} = Z = \frac{I}{Y_{max}}$$

$$= \frac{\pi (D^4 - d^4)}{64} \times \frac{2}{D} = \frac{\pi}{32}\left[\frac{(D^4 - d^4)}{D}\right] \quad ...(5.7)$$

$$\text{Moment of resistance, } M = \sigma Z = \sigma \times \frac{\pi}{32}\frac{(D^4 - d^4)}{D}$$

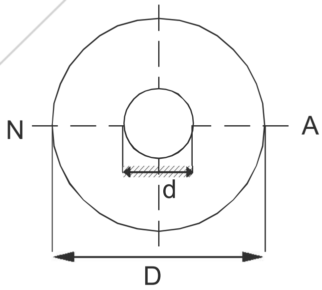


Fig. 5.6

<u>Example</u>

A 250mm (depth) x 150mm (width) rectangular beam is subjected to maximum bending moment of 750 KNm determine.

(i) The maximum stress in the beam.

(ii) If the value of E for the beam material is 200 GN/m^2.

Find out the radius of curvature for that portion of the beam where the bending is maximum.

(iii) The value of the longitudinal stress at a distance of 65mm from the top surface of the beam.

Solution : Refer to Fig 5.7

Width of the beam = b = 150 mm = 0.15m

Depth of the beam = d = 250 mm = 0.25m

Maximum bending moment M = 750KN.m

Young's modulus of elasticity, E = 200 GN/m2….

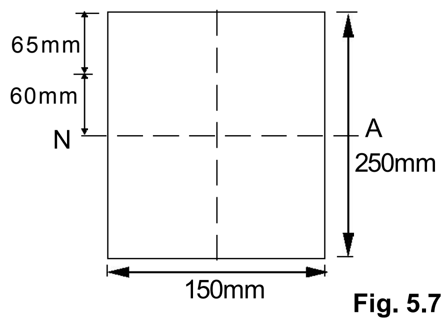


Fig. 5.7

(i) Maximum stress in the beam :

$$\text{Moment of inertia } I = \frac{bd^4}{12} = \frac{0.15 \times 0.25^3}{12} = 0.0001953 m^4$$

Distance of the neutral axis (N.A) from top surface of the beam

$$Y = \frac{d}{2} = \frac{0.25}{2} = 0.125 m$$

$$\text{using the relation } \frac{M}{I} = \frac{\sigma}{Y},$$

$$\text{we get } \sigma = \frac{M.Y}{I} = \frac{750 \times 10^3 \times 0.125}{0.0001953}$$

$$= 4.8 \times 10^8 N/mm2 = 480 MN/m^2$$

Hence the maximum stress in the beam $= 480 MN/m^2$ (Ans)

(ii) Radius of curvature, R:

Using the relation $\frac{M}{I} = \frac{E}{R}, R = \frac{EI}{M} = \frac{200 \times 10^9 \times 0.0001953}{750 \times 10^3} = 52.08 m$ (Ans)

(iii) Longitudinal stress at a distance of 65mm from top surface of the beam, using the relation $\frac{M}{I} = \frac{\sigma}{Y} = \frac{\sigma_1}{Y_1}$

$$\text{we get } \sigma 1 = \frac{MY_1}{I} = \frac{750 \times 10^3 \times (60 \times 10^{-3})}{0.0001953} \times 10^{-6} = MN/m^2$$

$$= 230.4 MN/m2 \text{ (Ans)}$$

CHAPTER 6.0

STRUT

A structural member subjected to an axial compressive force is called a strut.

Column

It is a vertical strut used in building or frame.

Axial load on column

The column fails by compressive stress.

The load, the least value of P which will cause the column to buckle, and it is called the Euler or crippling load.

The column in actual practice is subjected to following end conditions.

(1) Both ends hinged

(2) Both ends fixed

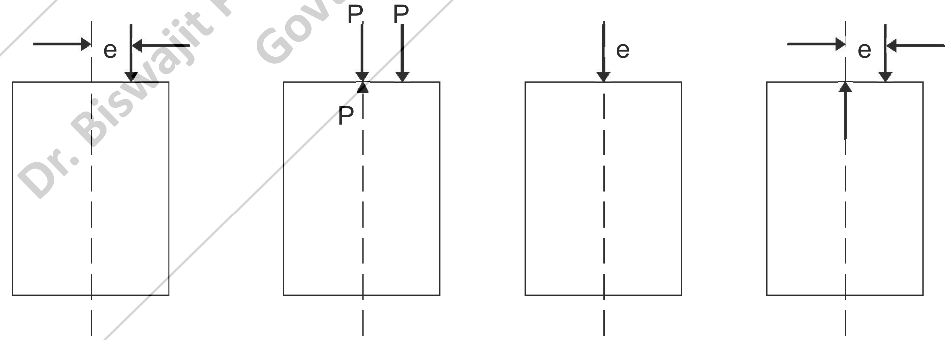
(3) One end is fixed and other end hinged.

(4) One end is fixed and other end free.

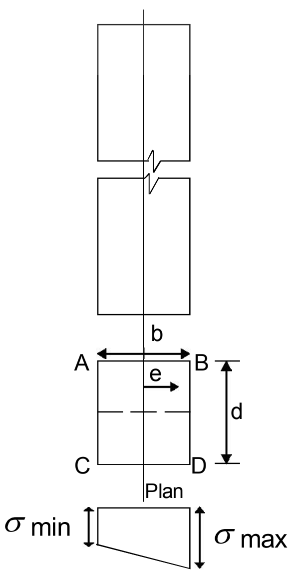
6.2 Eccentric load in columns

Eccentric load

A load whose line of action does not coincide with the axis of a column is called eccentric load.



Direct stresses, bending stresses, maximum & minimum stresses.



Consider the above column ABCD subjected to an eccentric load about

Let P = Load acting on the column

e = Eccentricity of the load

b= Width of the column section

d = Thickness of the column

Now Are of the section = bd

Moment of Inertia, I = $\frac{d.b^3}{12}$

Modulus of section, $Z = \frac{I}{y} = \frac{d.b^3/12}{b/12} = \frac{db^2}{12}$

Direct stress, $\sigma_0 = \frac{P}{A}$

Moment due to load, M = p.e

Bending stress at any point of column section at a distance y from y-y-axis

$$\sigma_b = \frac{M}{I}y = \frac{M}{Z}$$

or

$$\text{at } y = \frac{b}{2}$$

$$\sigma_b = \frac{M.\frac{b}{2}}{\frac{db^3}{2}} = \frac{6M}{db^3} = \frac{6p.e}{db^2} = \frac{6p.e}{A.b}$$

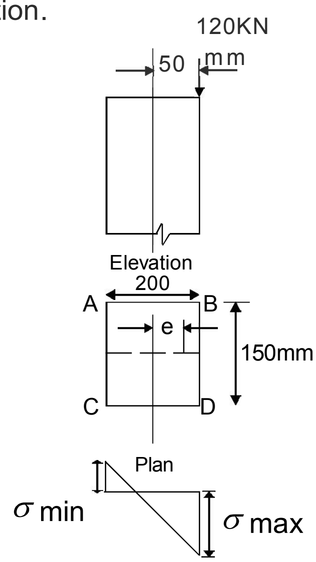
Total stress = direct stress + bending stress

$$= \frac{P}{A} \pm \frac{M}{Z} = \frac{P}{A} \pm \frac{6P.e}{Ab}$$

Problem

A rectangular column 200mm wide and 150mm thick is carrying a vertical load of 120KN at an eccentricity of 50mm in a plane bisecting the thickness determine the maximum and minimum intensities of stress in the section.

Solution



Given

b = 200mm, d = 150mm, p = 120KN, e = 50 mm

Maximum stress

A = b x d = 200 x 150 = 30,000 mm2

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right)$$

$$= \frac{120 \times 10^3}{30,000}\left(1 + \frac{6 \times 50}{200}\right)$$

$$= 10\,N/mm^2 = 10\,MPa \text{ (Ans)}$$

Minimum Stress

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)$$

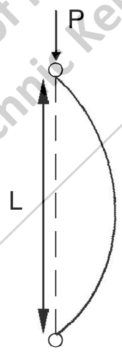
$$= \frac{120 \times 10^3}{30,000}\left(1 - \frac{6 \times 50}{200}\right)$$

$$= -2MPa \text{ (tension)}$$

6.4 Buckling load computation

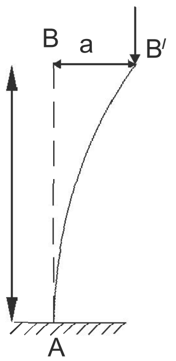
(1) Columns with both ends hinged

$$P = \frac{\pi^2 EI}{L^2}$$



(2) Columns with one end fixed and the other free

$$P = \frac{\pi^2 EI}{4L^2}$$



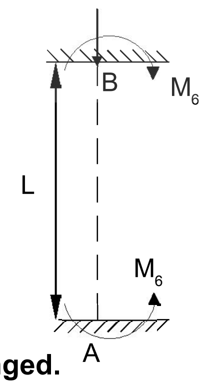
Cohers

E – Youngs modulus

I = Moment of Inertia about YY-axis.

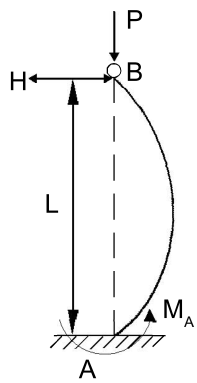
(3) Columns with both ends fixed.

$$P = \frac{\pi^2 EI}{L^2}$$



(4) Columns with one end fixed and the other hinged.

$$P = \frac{\pi^2 EI}{L^2}$$



CHAPATER 7.0

TORSION

7.1 Assumption of pure torsion

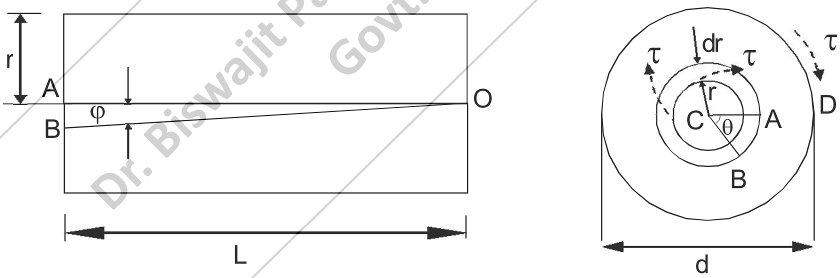
If a shaft is acted upon by a pure torque T about its polar axis, shear stress will be set up in directions perpendicular to the radius on all transverse sections. This is called as the shaft under torsion.

Following assumptions are made.

1. The material of the shaft is uniform through out
2. The twist along the shaft is uniform.
3. Normal cross sections of the shaft, which were plane and circular before the twist, remains plane and circular even after the twist.
4. All diameters of the normal cross section which were straight before the twist, remain straights with their magnitude unchanged, after the twist.

7.2 The torsion equation for solid shaft.

These above assumption is justified by the symmetry of the section.



The left hand figure shows the shear strain φ of elements at a distance r from the axis (φ is constant far constant T), so that a line originally OA twists to OB, and $\angle ACB = \theta$ the relative angle of twist of cross sections a distance L apart.

$$\text{Arc}\, AB = r\theta = L\varphi \text{(approx)}$$

$$\text{But } \varphi = \frac{\tau}{G}, \text{ where } G - \text{modulus of rigidity}$$

$$\text{or } \varphi = \frac{r.\theta}{L}$$

$$\frac{r.\theta}{e} = \frac{\tau}{G}$$

$$\text{or } \frac{\tau}{r} = \frac{G.\theta}{L}$$

The torque can be equated to the sum of the moments of the tangential stresses on the element $2\pi rdr$;

i.e. $T = \int \tau(2\pi r dr) r$

or, $T = \frac{G\theta}{L}.J$

Where J polar moment of inertial

$\frac{T}{J} = \frac{G\theta}{L}$

combing $\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$

for a solid shaft $J = \frac{\pi D^4}{32}$

and the max stress

$\tau_{max} = \frac{16T}{\pi D^3}$ at $r = \frac{D}{2}$

for a holloro shaft

$J = \frac{\pi}{32}(D^4 - d^4)$

and $\tau_{max} = \frac{16.D.T}{\pi(D^4 - d^4)}$ at $r = \frac{D}{2}$

Torsional stiffness, $K = \frac{T}{\theta} = \frac{GJ}{L}$

7.3 Comparison between solid and hollow shaft subjected to pure torsion.

Example

Compare the weights of equal lengths of hollow and solid shaft to transmit a given torque for the same maximum shear stress if the inside diameter is $\frac{2}{3}$ of the outside.

Solution

Nro, $\frac{T}{\tau} = \frac{2J}{D} = \frac{\pi D^3}{16}$ for solid shaft

and $\frac{T}{\tau} = \frac{\pi(D_1^4 - d^4)}{16D}$ for hollow shaft

or $\frac{T}{\tau} = \frac{\pi D_1^3}{16}\left(1 - \left(\frac{2}{3}\right)^4\right)$

$= \frac{65 \text{ x } \pi D1^3}{81\text{x}16}$

Equating these two shaft

$\frac{\pi D^3}{16} = \frac{65 \text{ x } \pi D1^3}{81\text{x}16}$

$D_1 = D.3\sqrt{81/65} = 1.075D$

$\boxed{D_1 = 1.075D}$

Ratio of weights of equal lengths

$$= (D_1^2 - d^2)/D^2$$

$$= (D_1/d)^2\left(1 - \frac{4}{9}\right)$$

$$= \left(\frac{5}{9}\right)2 x 1.075^2$$

$$= 0.642$$

Problem

A circular shaft of 50mm diameter is required to transmit torque from one shaft to another find the safe torque, which the shaft can transmit. If the $\tau = 40MPa$

Solution

$$D = 50 mm, \tau_{max} = 40MPa$$

we know

$$T = \frac{\pi}{16} x \tau D^3$$

$$= \frac{\pi}{16} x 40 x 50^3$$

$$= 0.982 x 10^6 N - mm$$

$$= 0.982 KN - m$$

STRENGTH OF MATERIALS

(SOM)

Chapter-01 :- Simple Stress & Strain

Load: Any external force acting on a body or machine part.

Types of Load:-

a) Dead or Steady load :- When the load doesn't change magnitude & direction.

b) Live or Variable load :- when the load changes continuously.

c) Suddenly applied or shock Loads :- when the load is suddenly applied or removed.

d) Impact Load :- When the load is applied with some initial velocity.

Stress :- The internal resistance of a body to the applied force is called stress. Mathematically,

$$\sigma = P/A,$$

$P \rightarrow$ force acting on the body

$A \rightarrow$ cross-sectional area of the body.

In SI units, 1 Pascal = 1 Pa = 1 N/m^2

1 MPa = 1 N/mm^2 & 1 GPa = 1 kN/mm^2

Strain :- The deformation undergone by a body due to the application of external force is known as strain.

$$\text{Strain, } \varepsilon = \frac{\delta l}{l},$$

$\delta l \rightarrow$ change in length

$l \rightarrow$ original length.

Tensile Stress & Strain :

When a body is subjected to two equal and opposite axial pulls, then tensile stress is induced in the body.

Due to the application of tensile load there will be decrease in cross-sectional area and an increase in the length of the body and the ratio between the change (increases) in length to original length is known as tensile strain.

Tensile stress, $\sigma_t = P/A$ & Tensile strain, $\varepsilon_t = \frac{\delta l}{l}$.

Compressive Stress & Strain :-

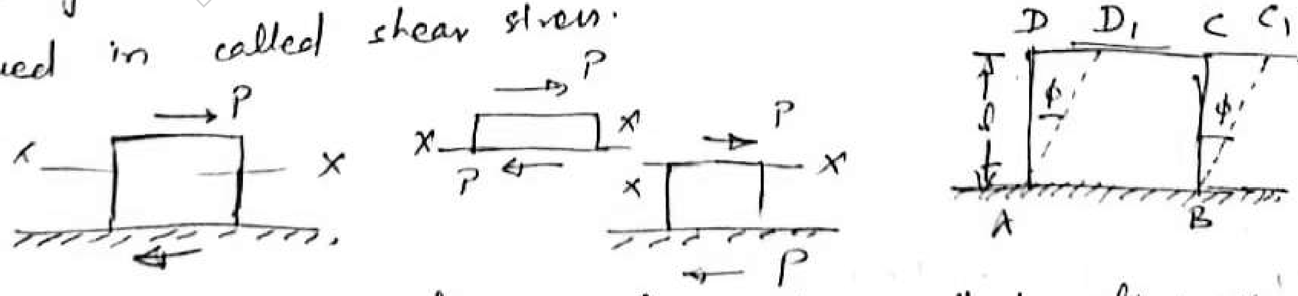
When a body is subjected to two equal and opposite axial pushes, then compressive stress is induced in the body.

Due to the application of compressive load there will be increase in cross-sectional area and an decrease in the length of the body and the ratio between the change (decreases) in length to original length is known as compressive strain.

Compressive stress, $\sigma_c = P/A$ & Compressive strain, $\varepsilon_c = \frac{\delta l}{l}$.

Shear Stress & Strain :-

When a section is subjected to two equal & opposite forces, acting tangentially across the resisting section, as a result of which the body tends to shear off across the section, the stress induced in called shear stress.



Due to the application of shear load there will be distortion in the body and the ratio between the deformation and original length is called shear strain.

Shear stress $= \frac{P}{AB}$. Shear strain $= \frac{dl}{l} = \frac{\text{Transverse displacement}}{\text{Distance from the lower face}}$

$\tau =$ $= \frac{dl}{l} = \phi$

SOM

Hooke's Law:-

→ It has been found that for a given section there is a limiting value of force upto and within which the deformation entirely disappears on the removal of force & the intensity of stress corresponding to this limiting force is called elastic limit of the material.

→ The law states that " when a material is loaded, within its elastic limit, the stress is proportional to the strain".

Mathematically,

$$\frac{\text{Stress}}{\text{strain}} = E = \text{Constant}$$

Young's Modulus or Modulus of Elasticity:-

Whenever a material is loaded within elastic limit the stress is ~~directly~~ proportional to strain.

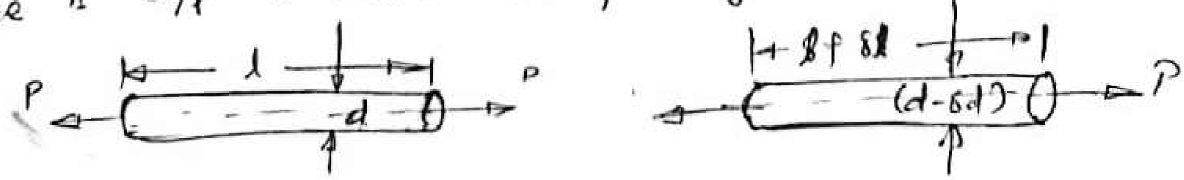
Mathematicall, $\sigma \propto \epsilon$

$= E \times \epsilon$.

E → a constant of proportionality known as modulus of elasticity or Young's Modulus and is that value of tensile stress which when applied to a uniform bar will increase its length to double the original length if the material ~~would~~ remain perfectly elastic throughout of the bar such an extensile strain.

Primary or Linear Strain / Secondary or Lateral Strain

The deformation of the bar per unit length in the direction of force is δl/l is known as primary or linear strain.



This will be followed by the decrease in the diameter from d to (d - δd).

→ So every direct stress is always accompanied by a strain in its own direction & an opposite kind of strain in every direction at right angles to it and this strain is called.

secondary / lateral strain = $\frac{\delta d}{d}$.

Poisson's Ratio :-

When a body is elastically stressed the ratio between the lateral strain and linear strain is constant and mathematically,

$$\frac{\text{Lateral strain}}{\text{Linear strain}} = \text{constant.}$$

$$= \text{Poisson's ratio} = \frac{1}{m} \text{ or } \mu.$$

$$\text{Lateral strain} = \frac{1}{m} \times \epsilon = \mu\epsilon.$$

Steel → $\mu = 0.25$

Rubber → $\mu = 0.5$

Volumetric Strain :-

Whenever a body is subjected to a single force or a system of forces it undergoes some changes in its dimensions due to which the volume changes. So mathematically,

$$\frac{\text{Change in Volume}}{\text{Original Volume}} = \text{Volumetric strain } (\epsilon_v)$$

$$\Rightarrow \epsilon_v = \frac{\delta V}{V}$$

Bulk Modulus :-

When the body is subjected to three mutually perpendicular stresses of equal intensity the ratio of direct stress to the corresponding volumetric strain is known as bulk modulus.

$$\text{Bulk modulus, } K = \frac{\text{Direct Stress}}{\text{Volumetric Strain}} = \frac{\sigma}{(\delta V/V)}$$

SOM

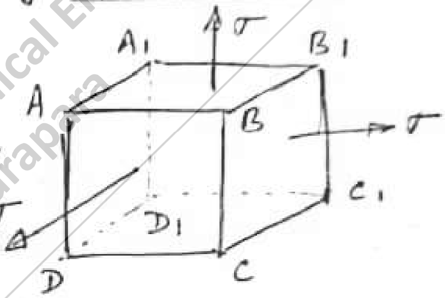
Shear Modulus or Modulus of Rigidity :

Within elastic limit, it has been experimentally found that, the shear stress is proportional to shear strain and Mathematically,

$$\tau \propto \phi$$
$$= C\phi \Rightarrow \boxed{C = \frac{\tau}{\phi}}$$

= a constant of proportionality known as shear modulus or modulus of rigidity.

Relationship between Bulk modulus & Young's modulus :-



Consider a cube ABCD A₁B₁C₁D₁ which is subjected to three mutually perpendicular tensile stresses, σ of equal intensity.

l → length of the cube

E → Young's modulus of block material.

The strains occured in the side AB of the cube are,

a) Tensile strain equal to (σ/E) due to stresses on faces BB₁CC₁ & AA₁DD₁

b) Compressive lateral strain due to stresses on faces AA₁BB₁ & DD₁CC₁

c) Compressive lateral strain equal to $\frac{1}{m} \times \frac{\sigma}{E}$ due to stresses on faces ABCD & A₁B₁C₁D₁.

So the net tensile strain suffered by the side AB,

$$\frac{\delta l}{l} = \frac{\sigma}{E} - \left(\frac{1}{m} \times \frac{\sigma}{E}\right) - \left(\frac{1}{m} \times \frac{\sigma}{E}\right) = \frac{\sigma}{E}\left(1 - \frac{2}{m}\right)$$

We have the original volume of the cube,

$$V = l^3$$

$$\therefore \frac{\delta V}{\delta l} = 3l^2 \Rightarrow \delta V = 3l^2 \delta l = 3l^3 \times \frac{\delta l}{l}$$

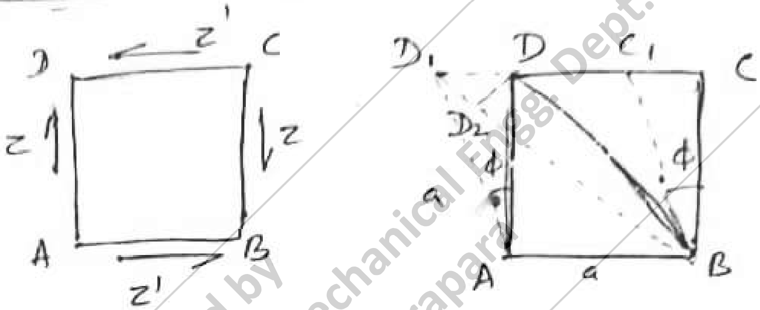
$$\Rightarrow \delta V = 3l^3 \times \frac{\sigma}{E}\left(1 - \frac{2}{m}\right)$$

$$\therefore \frac{\delta V}{V} = \frac{3l^3}{l^3} \times \frac{\sigma}{E}\left(1 - 2/m\right) = \frac{3\sigma}{E}\left(1 - 2/m\right)$$

$$\Rightarrow E = 3 \frac{\sigma}{(\delta V/V)} \left(1 - 1/m\right)$$

$$\Rightarrow \boxed{E = 3K\left(1 - 2/m\right)} \quad \text{or} \quad \boxed{K = \frac{mE}{3(m-2)}}$$

Relationship between Young's Modulus and Shear Modulus :



Strain in the length BD = Strain in the length of BD due to diagonal tensile stresses on the plane AC + Strain in length of BD due to diagonal compressive stresses on the plane BD

$$\text{Strain on } BD = \frac{\tau}{E} + \frac{\tau}{mE} = \frac{\tau}{E}\left(1 + \frac{1}{m}\right)$$

Increase in length of the diagonal $BD = BD_1 - BD$.

Let $DD_2 \perp BD_1$.

Since $\angle DBD_2$ is very small, $BD = BD_2$

$BD_1 - BD = D_1D_2$

So Increase in length of $BD = DD_1 \cos DD_1D_2$

But $\angle DD_1D_2 \cong \angle CDB = 45^\circ$

$$\Rightarrow D_1D_2 = DD_1 \cos 45^\circ = \frac{DD_1}{\sqrt{2}}$$

But shear strain, $\phi = \frac{DD_1}{AD} = \frac{DD_1}{a} \Rightarrow DD_1 = a\phi$

$$\Rightarrow D_1D_2 = a\phi/\sqrt{2}$$

Length of the diagonal, $BD = a\sqrt{2}$

$$\therefore \text{Strain} = \frac{\text{Increase in length}}{\text{Original length}} = \frac{(a\phi/\sqrt{2})}{(a\sqrt{2})} = \frac{\phi}{2} = \frac{\tau}{E}\left(1 + \frac{1}{m}\right)$$

$$\Rightarrow E = \frac{2\tau}{\phi}\left(1 + \frac{1}{m}\right) \Rightarrow \boxed{E = 2C\left(1 + \frac{1}{m}\right)}$$

$$\Rightarrow \boxed{C = \frac{mE}{2(m+1)}}$$

$$\left(1+\frac{1}{m}\right) = \frac{E}{2C} \quad (1) \qquad \& \qquad \left(1-\frac{2}{m}\right) = \frac{E}{3K} \quad (2)$$

(Eqⁿ (1) × 2) + (Eqⁿ (2))

$$\Rightarrow 3 = \frac{E}{C} + \frac{E}{3K} = \frac{E(3K+C)}{3KC} \Rightarrow \boxed{E = \frac{9KC}{3K+C}}$$

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~~Q-11 An elastic rod~~

Principle of Superposition :-

The resulting deformation of a body due to the application of number of forces acting at different sections is equal to ~~the resulting deformation of~~ the algebraic sum of the deformations of the individual sections.

The resulting deformation,

$$\delta l = \frac{Pl}{AE} = \frac{1}{AE}\left[P_1 l_1 + P_2 l_2 + \cdots\right]$$

$P_1 \rightarrow$ force acting on section-1

$l_1 \rightarrow$ length of section-1.

Stresses in composite sections :

Suppose the cross-section of a member consists of different materials the load applied on the member will be shared by the various components of the section.

Total load on the structure = load on outer tube + load on the inner tube.

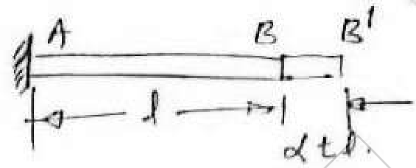
Temperature Stresses :-

→ When the temperature of a material changes there will be corresponding change in its ~~temperature~~ dimensions.

→ When a member is free to expand or contract due to the rise or fall of temperature, no stresses will be induced in the member.

→ But, if the natural change in length due to rise or fall of temperature be prevented, stress will be offered.

Increase in length due to increase in temperature, $\delta l = \alpha t l$



(If the ends of the bar are fixed or rigid supports, so that its expansion is prevented, then compressive strain induced in the bar)

$$\text{So strain} = \varepsilon = \frac{\delta l}{l} = \frac{\alpha t l}{l} = \alpha t$$

$\therefore$ Stress, $\sigma = \varepsilon E = \alpha t E$ (Temperature stress) → when the rods are prevented for expanding.

$$\text{In general temperature strain} = \frac{\text{Expansion or Contraction prevented}}{\text{Original length}}$$

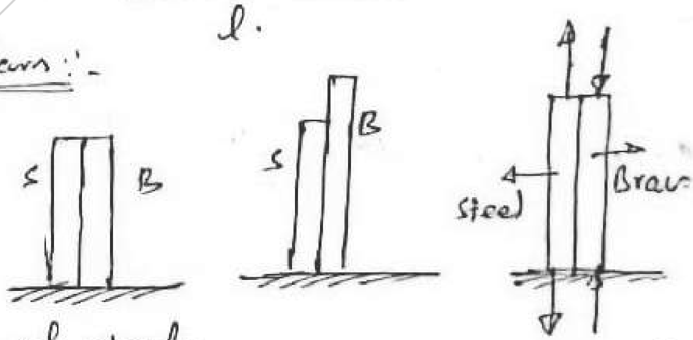
For a rod of length l, when subjected to rise in temperature, is permitted to expand only by δ,

$$\text{the temperature strain } e = \frac{\text{Expansion Prevented}}{\text{Original length}} = \frac{\alpha t l - \delta}{l}$$

$$\therefore \text{temperature stress} = \sigma = \varepsilon E = \frac{E(\alpha t l - \delta)}{l}$$

Thermal Stresses in Composite Bars :-

→ As co-efficient of linear expansion of the brass is greater than that of steel, free expansion of the brass will be more than that of steel.



→ Since both the members are not free to expand, therefore the expansion of the composite bar as a whole, will be less than that of the brass but more than that of the steel.

→ So the brass will be subjected to compressive force, whereas the steel will be subjected to tensile force.

$\sigma_1, \varepsilon_1, \alpha_1, A_1$ → Stress, strain, co-efficient of linear expansion, cross-sectional area of brass

$\sigma_2, \varepsilon_2, \alpha_2, A_2$ → Corresponding values for steel

ε → Actual strain of the composite bar

As the compressive load on the brass is equal to the tensile load on the steel,

$$\sigma_1 A_1 = \sigma_2 A_2$$

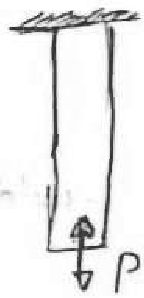
$$\varepsilon_B = \alpha_1 t - E \quad \& \quad \varepsilon_S = \alpha_2 t - E$$

$$\varepsilon_1 + \varepsilon_2 = -t(\alpha_1 + \alpha_2)$$

Strain Energy:-

→ As the member is stretched by the load, the strains produced in the member, increase the energy level of the member itself. By the principle of conservation of energy, the work done by the load in stretching the member is equal to the energy absorbed or stored by the member.

→ Strain energy absorbed or stored by the member when work is done is called strain energy or resilience.



Work done against resistance on the member

= Strain energy stored by the member

= Average Resistance × displacement $= \frac{R}{2} \times \delta$

R → is the resistance offered by the member against the extension.

∴ Stress $= \sigma = \frac{R}{A} \Rightarrow R = \sigma A = P$

P → applied load.

but $\delta = E l$ & $R = \sigma A$

∴ Strain energy stored by the member = Work done

= Force × Displacement $= \frac{\sigma A}{2} \times e l$

$= \frac{1}{2} \times \sigma \times E \times (A.l)$

$= \frac{1}{2} \times$ Stress × Strain × Volume of the member.

$= \frac{1}{2} \times \sigma \times \frac{\sigma}{E} \times Al = \frac{\sigma^2}{2E}(Al)$

Strain energy density : Strain energy stored per unit volume

$$= \frac{\sigma^2}{2E}$$

Stress due to Various types of Axial Loads :

a) Gradually Applied load :- Let a load of magnitude P be applied axially on a member of length l and uniform cross-sectional area A. , $\delta l \rightarrow$ extension of the rod, $\sigma \rightarrow$ stress intensity.

Strain Energy stored by the member = $\frac{\sigma^2}{2E} \times Al$

Workdone by the external load = Average load × extension

$$= \frac{1}{2} \times P \times \delta l$$

Equating the above,

$$\frac{\sigma^2}{2E} \times Al = \frac{1}{2} \times P \times \delta l$$

$$= \frac{1}{2} \times P \times \frac{\sigma}{E} \times l$$

$$\Rightarrow \sigma = P/A$$

[As the load applied is gradual, and varies from 0 to P, therefore the average load is equal to P/2]

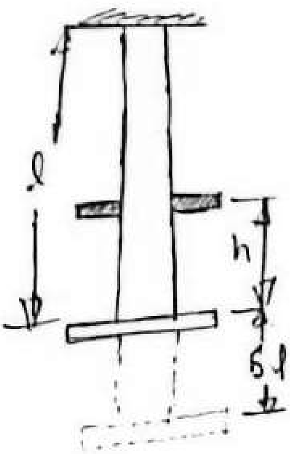
b) Suddenly Applied Load : Since the load is applied suddenly, therefore the load (P) is constant throughout the process of deformation of the bar.

Equating the strain energy stored by the member to the workdone.

$$\frac{\sigma^2}{2E} \times Al = P \times \delta l = P \times \frac{\sigma}{E} l$$

$$\Rightarrow \sigma = 2 P/A$$

c) Impact Load :- Equating the loss of PE to the strain energy stored by the rod.



$$P(h + \delta l) = \frac{\sigma^2}{2E} \times Al$$

$$\Rightarrow P\left(h + \frac{\sigma l}{E}\right) = \frac{\sigma^2}{2E} \times Al.$$

$$\frac{\sigma^2}{2E} \times Al - \frac{P\sigma l}{E} = Ph$$

$$\Rightarrow \sigma^2 - 2\frac{P\sigma}{A} = \frac{2PhE}{Al}$$

Adding $\frac{P^2}{A^2}$ to both sides,

$$\Rightarrow \sigma^2 - \frac{2P\sigma}{A} + \frac{P^2}{A^2} = \frac{P^2}{A^2} + \frac{2PhE}{Al}$$

$$\Rightarrow \left(\sigma - P/A\right)^2 = \frac{P^2}{A^2} + \frac{2PhE}{Al}$$

$$\Rightarrow \sigma = \frac{P}{A} + \sqrt{\frac{P^2}{A^2} + \frac{2PhE}{Al}}$$

when δl is very small in comparison with h.

loss of PE = Ph

Equating the loss PE to the strain energy.

$$Ph = \frac{\sigma^2}{2E} \times Al \Rightarrow \sigma = \sqrt{\frac{2PhE}{Al}}$$

Resilience

→ It is the ability of a material to absorb energy when it is deformed elastically, and release that energy upon unloading.

→ Proof resilience is defined as the maximum energy that can be absorbed upto the elastic limit, without creating a permanent distortion.

→ The modulus of resilience is defined as the maximum energy that can be absorbed per unit volume without creating a permanent distortion.

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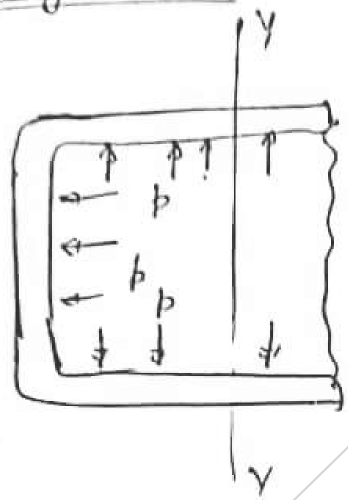
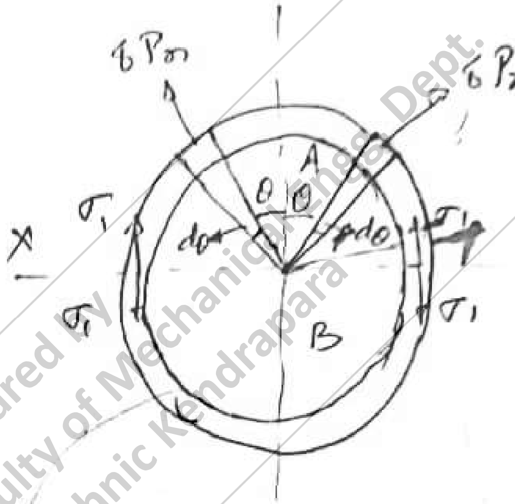
Chapter: 02

THIN CYLINDER AND SPHERICAL SHELL UNDER INTERNAL PRESSURE

Thin Cylinders:





Let us consider a cylindrical shell whose
$d \to$ internal diameter, $l \to$ length of the shell
$t \to$ thickness $p \to$ pressure intensity, $r \to$ radius of shell.

Let us consider two elementary strips subtending an angle $d\theta$ at an angle θ on either side of the vertical through the centre.

Normal force on each strip,

$$dP_n = p\, r\, d\theta\, l$$

The resultant of the two normal forces on the two ~~∴ Total force nor~~ elemental strips $= dP = 2prl\, d\theta \cos\theta$ acting vertically.

∴ Total force normal to XX on one side of XX

= Total bursting force $= P_1$

$$= \int_0^{\pi/2} 2prl \cos\theta\, d\theta = 2prl = pdl$$

= Intensity of radial pressure × projected area

If σ_1 is the intensity of tensile stress induced in the metal across the section XX.

The resisting force offered by the section $XX = \sigma_1 \times 2lt$.

Equating the resisting force to the bursting force,

$$\sigma_1 \times 2 \times l \times t = p\,d\,l \Rightarrow \boxed{\sigma_1 = \frac{pd}{2t}}$$

The direction of this tensile stress is along the circumference of the shell and a stress so induced is called a <u>hoop or circumferential stress</u>.

Let us now consider a section Y-Y normal to the axis of the shell. So the force acting on the end of the shell,

$$P = p\frac{\pi d^2}{4}$$

So the section YY will offer a resistance. If σ_2 is the tensile stress on the section.

Equating the resistance offered by the section YY to the total force on one end of the shell.

$$\sigma_2 \pi d t = p\frac{\pi d^2}{4} \Rightarrow \boxed{\sigma_2 = \frac{pd}{4t}} \rightarrow \text{Longitudinal Stress.}$$

Circumferencial Strain $= \varepsilon_1 = \frac{\sigma_1}{E} - \frac{\sigma_2}{mE}$

$$\varepsilon_1 = \frac{pd}{2tE} - \frac{pd}{4tEm} \Rightarrow \boxed{\varepsilon_1 = \frac{pd}{4tE}\left(2 - \frac{1}{m}\right)}$$

Longitudinal Strain $= \varepsilon_2 = \frac{\sigma_2}{E} - \frac{\sigma_1}{mE} = \frac{pd}{4tE} - \frac{pd}{2tEm}$

$$\boxed{\varepsilon_2 = \frac{pd}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)}$$

Circumferential Strain $= \frac{\text{Change in Circumference}}{\text{Original circumference}} = \frac{\pi \delta d}{\pi d} = \varepsilon_1$

$$\Rightarrow \boxed{\varepsilon_1 = \frac{\delta d}{d}} = \text{strain of the diameter.}$$

Longitudinal Strain $= \frac{\text{Change in length}}{\text{Original length}} = \varepsilon_2$

$$\Rightarrow \boxed{\varepsilon_2 = \frac{\delta l}{l}}$$

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Capacity of shell = $V = \frac{\pi}{4} d^2 l$ = Volume.

∴ Change in capacity or volume of the shell,

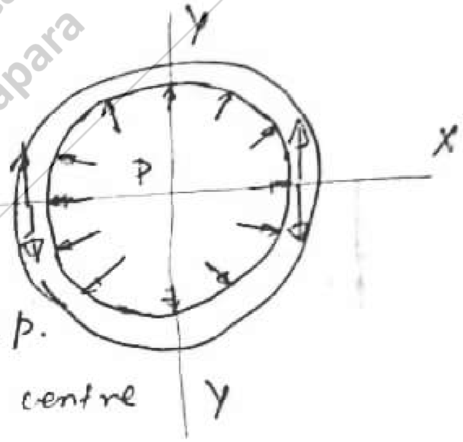
$$\delta V = \frac{\pi}{4} d^2 \delta l + \frac{\pi}{4} \cdot 2d\, \delta d\, l$$

$$\Rightarrow \frac{\delta V}{V} = \frac{\delta l}{l} + 2\frac{\delta d}{d} = \varepsilon_2 + 2\varepsilon_1$$

$$= \frac{pd}{2tE}\left[\frac{1}{2} - \frac{1}{m}\right] + 2\frac{pd}{2tE}\left(1 - \frac{1}{2m}\right) = \frac{pd}{2tE}\left(\frac{5}{2} - \frac{2}{m}\right)$$

$$\Rightarrow \boxed{\frac{\delta V}{V} = \frac{pd}{2tE}\left[\frac{5}{2} - \frac{2}{m}\right]} \rightarrow \underline{\text{Volumetric Strain}}$$

Thin Spherical Shells



Let us consider a thin spherical shell of internal diameter 'd' and thickness 't' and subjected to a net internal pressure p.

Let us consider a section XX through the centre of the shell.

Bursting force P = p × projected area = $p \times \frac{\pi d^2}{4}$

If σ_1 be the tensile stress induced on the section of the metal at the section XX.

∴ Resisting force = $\sigma_1 \pi d t$

Equating the resisting force to the bursting force.

$$\sigma_1 \pi d t = p\frac{\pi d^2}{4} \Rightarrow \boxed{\sigma_1 = \frac{pd}{4t}} \rightarrow \underline{\text{Hoop Stress}}$$

Similarly for any other section through the centre of the shell, the tensile stresses for the section will be,

$$\boxed{\sigma_2 = \frac{pd}{4t}} \rightarrow \underline{\text{Longitudinal Stress}}$$

In case of the thin spherical shell, the principal stresses σ_1 & σ_2 at any point are equal and alike.

So the strain in any direction,

$$= \varepsilon = \frac{\sigma_1}{E} - \frac{\sigma_2}{mE} = \frac{\sigma_1}{E}\left(1-\frac{1}{m}\right) = \frac{pd}{4tE}\left(1-\frac{1}{m}\right)$$

$$\Rightarrow \boxed{\varepsilon = \frac{pd}{4tE}\left(1-\frac{1}{m}\right)} \rightarrow$$ Hoop strain, Longitudinal strain

The increase in diameter due to internal pressure is given by,

$$\frac{\delta d}{d} = \varepsilon = \frac{pd}{4tE}\left(1-\frac{1}{m}\right) \Rightarrow \boxed{\delta d = \frac{pd^2}{4tE}\left(1-\frac{1}{m}\right)}$$

Original Volume $= V = \frac{4}{3}\pi r^3 \Rightarrow V = \frac{\pi d^3}{6}$

$$\Rightarrow \delta V = \frac{3\pi d^2}{6}\delta d \Rightarrow \frac{\delta V}{V} = 3\frac{\delta d}{d} = 3\varepsilon$$

$$\Rightarrow \boxed{\frac{\delta V}{V} = \frac{3pd}{4tE}\left(1-\frac{1}{m}\right)} \rightarrow$$ Volumetric Strain.

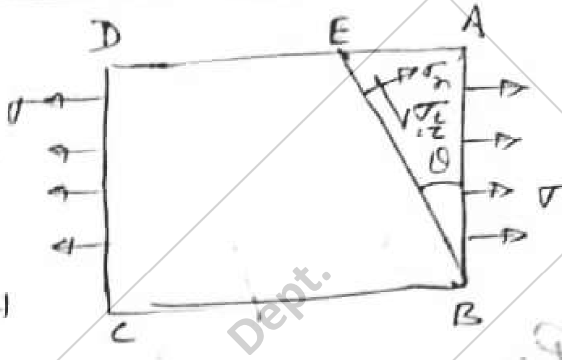
$$\Rightarrow \boxed{\delta V = \frac{\pi p d^4}{8tE}\left(1-\frac{1}{m}\right)}$$

Chapter: 03

TWO DIMENSIONAL STRESS SYSTEM

Stresses on Oblique Planes

Case-1: Member Subjected to Axial Load



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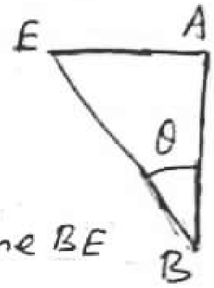
Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to a direct stress.

Consider any oblique plane BE at an angle θ to the cross-section on which normal and tangential stresses are induced.



From triangle AEB,

$$\cos\theta = \frac{BA}{BE} \Rightarrow BE = BA\sec\theta$$

For sectional area of the member along the plane BE

$$= A\sec\theta$$

Normal stress on the plane BE,

$$= \sigma_n = \frac{\text{Total force normal to the plane BE}}{\text{Sectional area along the plane BE}}$$

$$\Rightarrow \sigma_n = \frac{\sigma A\cos\theta}{A\sec\theta} \Rightarrow \boxed{\sigma_n = \sigma\cos^2\theta}$$

Tangential or shear stress on the plane BE.

$$= \sigma_t = \frac{\text{Tangential force along the plane BE}}{\text{Sectional area along the plane BE}}$$

$$\Rightarrow \sigma_t = \frac{\sigma A\sin\theta}{A\sec\theta} = \sigma\sin\theta\cos\theta$$

$$\Rightarrow \boxed{\sigma_t = \frac{\sigma}{2}\sin 2\theta} \rightarrow \tau$$

Resultant Stress, $\sigma_r = \sqrt{\sigma_n^2 + \sigma_t^2}$

$$= \sqrt{\sigma^2\cos^4\theta + \sigma^2\sin^2\theta\cos^2\theta}$$

$$\Rightarrow \boxed{\sigma_r = \sigma\cos\theta}$$

$\sigma_n (max) = \sigma \cos^2 0° = \sigma$ (for $\theta = 0°$)

$\sigma_n (min) = \sigma \cos^2 90° = 0$ (for $\theta = 90°$)

$\sigma_t (max) = \frac{\sigma}{2} \sin 2\theta = \frac{\sigma}{2}$ (for $2\theta = 90°$)

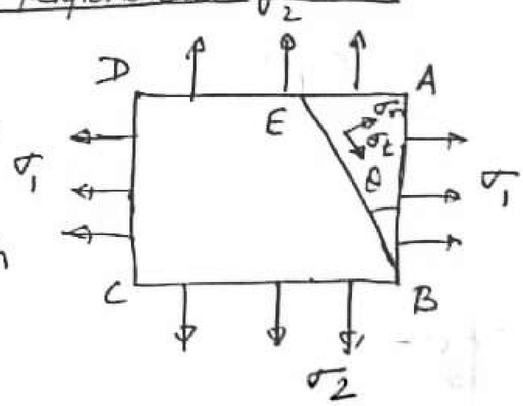
$\sigma_t (min) = \frac{\sigma}{2} \sin 0 = 0$ (for $2\theta = 0°$)

Principal Planes:- It has been observed that at any point in a strained material, there are three planes, mutually perpendicular to each other, which carries direct stresses only. Such a plane on which no tangential stress occurs is called principal plane.

Principal Stresses:- The magnitude of direct stress across a principal plane is known as principal stress.

Case:-2:- Member subjected to like direct or principal stresses in two mutually perpendicular directions

Consider a rectangular block ABCD which is subjected to principal stresses σ_1 & σ_2. Consider any oblique sectional plane BE at an angle θ with the principal plane BA.



Normal stress on the plane BE.

$$\sigma_n = \frac{\text{Total force normal to the plane BE}}{\text{Sectional area along the plane.}}$$

$$= \frac{\sigma_1 BA \cos\theta + \sigma_2 EA \sin\theta}{BE}$$

$$\Rightarrow \sigma_n = \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta = \frac{\sigma_1}{2}(1+\cos 2\theta) + \frac{\sigma_2}{2}(1-\cos 2\theta)$$

$$\Rightarrow \boxed{\sigma_n = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2}\cos 2\theta}$$

Tangential stress on the plane BE

$$= \sigma_t = \frac{\text{Total force tangential or parallel to the plane BE}}{\text{Sectional area along the plane BE}}$$

$$= \frac{\sigma_1 BA \sin\theta - \sigma_2 EA \cos\theta}{BE} = \sigma_1 \cos\theta \sin\theta - \sigma_2 \sin\theta \cos\theta$$

$$\Rightarrow \boxed{\sigma_t = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta}$$

Resultant Stress $= \sigma = \sqrt{(\sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta)^2 + (\sigma_1 - \sigma_2)^2 \sin^2\theta \cos^2\theta}$

$$\Rightarrow \boxed{\sigma = \sqrt{\sigma_1^2 \cos^2\theta + \sigma_2^2 \sin^2\theta}}$$

The angle that the line of action of the resultant stress makes with the normal to the plane is called the obliquity (ϕ).

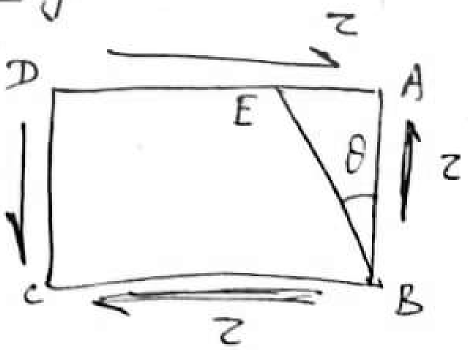
$$\Rightarrow \boxed{\tan\phi = \frac{\sigma_t}{\sigma_n}}$$

For σ_t to be maximum, $\theta = 45°$ or $135°$.

$$\Rightarrow \text{Maximum shear stress} = \boxed{\sigma_{t(max)} = \frac{\sigma_1 - \sigma_2}{2}}$$

Due to application of pure shear loading :-

Let us consider a rectangular block ABCD in a state of simple shear under the action of shear stress of intensity τ.



The normal stress $= \sigma_n = \frac{\tau AB \sin\theta + \tau EA \cos\theta}{BE}$

$$= \tau \sin\theta \cos\theta + \tau \cos\theta \sin\theta.$$

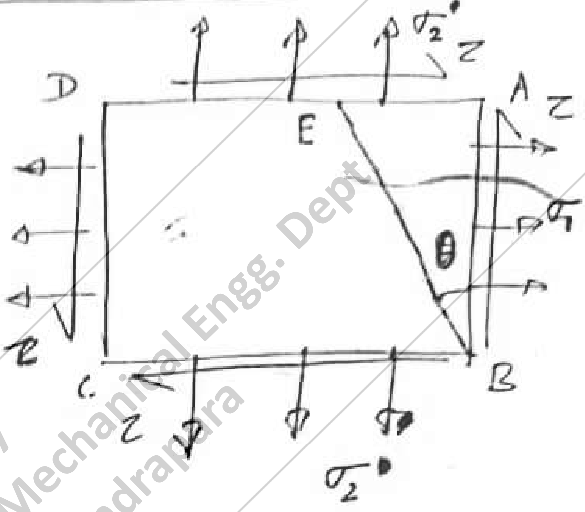
$$\Rightarrow \boxed{\sigma_n = \tau \sin 2\theta}$$

The tangential stress $= \sigma_t = \frac{\tau AB \cos\theta - \tau EA \sin\theta}{BE}$

$= \tau\cos^2\theta - \tau\sin^2\theta$

$\Rightarrow \boxed{\sigma_t = \tau\cos 2\theta}$

Due to the normal and tangential stresses on two mutually perpendicular planes:-



Let us consider a rectangular block ABCD which is subjected to normal stresses σ_1 & σ_2' and the tangential stress τ.

Consider a section plane BE at an angle θ with the plane AB.

Normal stress on the plane BE,

$$\sigma_n = \frac{\sigma_1 AB\cos\theta + \sigma_2' AE\sin\theta + \tau AE\cos\theta + \tau AB\sin\theta}{BE}$$

$$= \sigma_1\cos^2\theta + \sigma_2'\sin^2\theta + \tau\sin\theta\cos\theta + \tau\cos\theta\sin\theta$$

$$= \sigma_1\cos^2\theta + \sigma_2'\sin^2\theta + 2\tau\sin\theta\cos\theta$$

$$= \frac{\sigma_1}{2}(1+\cos 2\theta) + \frac{\sigma_2'}{2}(1-\cos 2\theta) + \tau\sin 2\theta$$

$$\Rightarrow \boxed{\sigma_n = \frac{\sigma_1+\sigma_2'}{2} + \frac{\sigma_1-\sigma_2'}{2}\cos 2\theta + \tau\sin 2\theta}$$

Tangential stress on the plane BE,

$$\sigma_t = \frac{\sigma_1 AB\sin\theta - \sigma_2' AE\cos\theta + \tau AE\sin\theta - \tau AB\cos\theta}{BE}$$

$$= \sigma_1\cos\theta\sin\theta - \sigma_2'\sin\theta\cos\theta + \tau\sin^2\theta - \tau\cos^2\theta$$

$$\boxed{\sigma_t = \frac{\sigma_1-\sigma_2'}{2}\sin 2\theta - \tau\cos 2\theta}$$

In order a plane may be a principal plane, the tangential stress on the plane must be zero:

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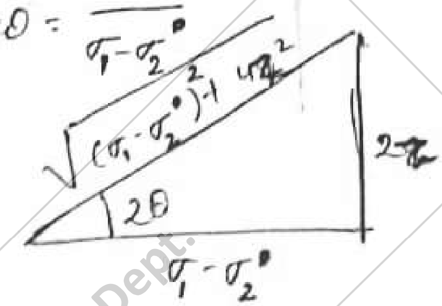
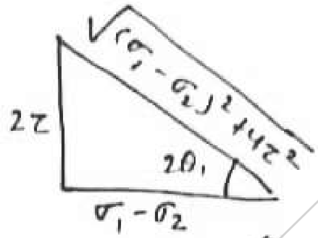
$$\Rightarrow \frac{\sigma_1 - \sigma_2'}{2} \sin 2\theta - \tau \cos 2\theta = 0 \Rightarrow \tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2'}$$

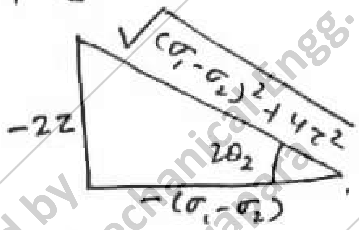


we have,



$$\sin 2\theta_1 = \frac{2\tau}{\sqrt{(\sigma_1-\sigma_2')^2+4\tau^2}}$$

$$\cos 2\theta_1 = \frac{\sigma_1-\sigma_2'}{\sqrt{(\sigma_1-\sigma_2')^2+4\tau^2}}$$



$$\sin 2\theta_2 = \frac{-2\tau}{\sqrt{(\sigma_1-\sigma_2')^2+4\tau^2}} \quad \& \quad \cos 2\theta_2 = -\frac{\sigma_1-\sigma_2'}{\sqrt{(\sigma_1-\sigma_2')^2+4\tau^2}}$$

The principal stresses σ_{p_1} & σ_{p_2} can be calculated by,

$$\sigma_{p_1} = \frac{\sigma_1+\sigma_2'}{2} + \frac{\sigma_1-\sigma_2'}{2}\cos 2\theta_1 + \tau \sin 2\theta_1$$

$$= \frac{\sigma_1+\sigma_2'}{2} + \frac{\sigma_1-\sigma_2'}{2}\frac{(\sigma_1-\sigma_2')}{\sqrt{(\sigma_1-\sigma_2')^2+4\tau^2}} + \frac{2\tau}{\sqrt{(\sigma_1-\sigma_2')^2+4\tau^2}}$$

$$= \frac{\sigma_1+\sigma_2'}{2} + \frac{1}{2\sqrt{(\sigma_1-\sigma_2')^2+4\tau^2}}\left[(\sigma_1-\sigma_2')^2+4\tau^2\right]$$

$$= \frac{\sigma_1+\sigma_2'}{2} + \frac{1}{2}\sqrt{(\sigma_1-\sigma_2')^2+\tau^2}$$

$$\Rightarrow \boxed{\sigma_{p_1} = \frac{\sigma_1+\sigma_2'}{2} + \sqrt{\left(\frac{\sigma_1-\sigma_2'}{2}\right)^2+\tau^2}}$$

$$\sigma_{p_2} = \frac{\sigma_1+\sigma_2'}{2} + \frac{\sigma_1-\sigma_2'}{2}\cos 2\theta_2 + \tau \sin 2\theta_2$$

$$= \frac{\sigma_1+\sigma_2'}{2} - \frac{\sigma_1-\sigma_2'}{2}\frac{(\sigma_1-\sigma_2')}{\sqrt{(\sigma_1-\sigma_2')^2+4\tau^2}} - \frac{2\tau^2}{\sqrt{(\sigma-\sigma')^2+4\tau^2}}$$

$$= \frac{\sigma_1+\sigma_2'}{2} - \frac{1}{2\sqrt{(\sigma_1-\sigma_2')^2+4\tau^2}}\left[(\sigma_1-\sigma_2')^2+4\tau^2\right]$$

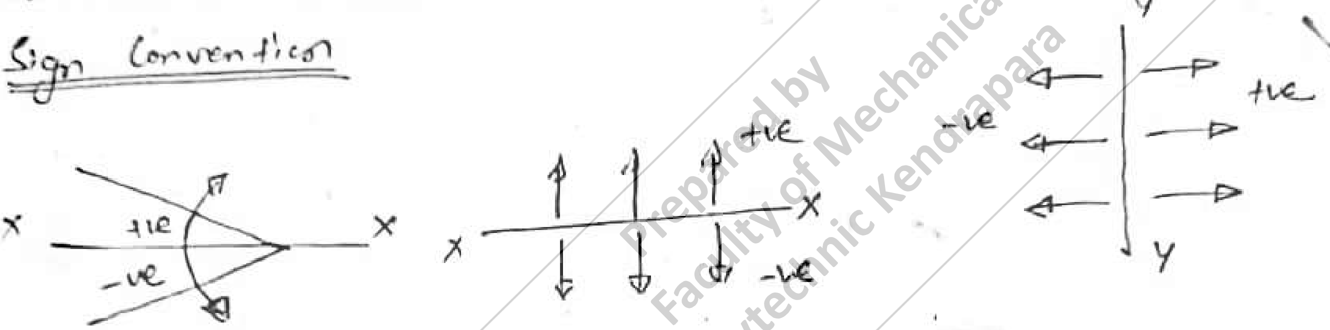
$$= \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

$$\Rightarrow \boxed{\sigma_{p_2} = \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}}$$

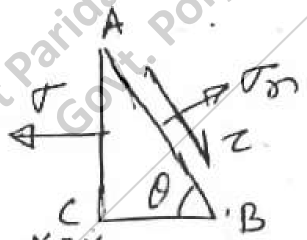
Greatest shear stress = $\boxed{\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}}$

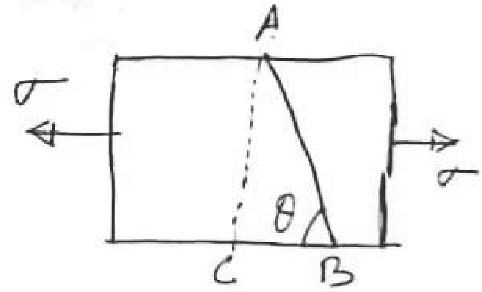
Graphical Method (Mohr's Circle) by a German Scientist. Otto Mohr in 1982

Sign Convention



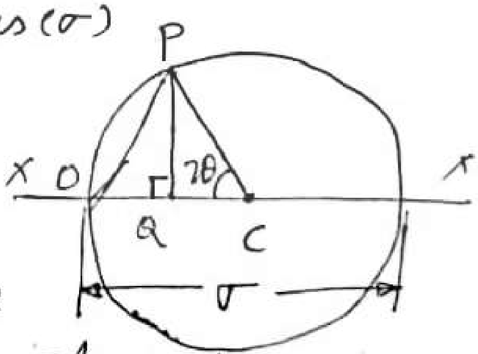
Case-1 :-





→ Take some suitable point O & through it draw a horizontal line XOX.

→ Cut off OJ equal to the tensile stress (σ) to some suitable scale and towards right (because σ is tensile). Bisect OJ at C, Now the point O represents the stress system on plane BC and the point J represents the stress system on plane AC.



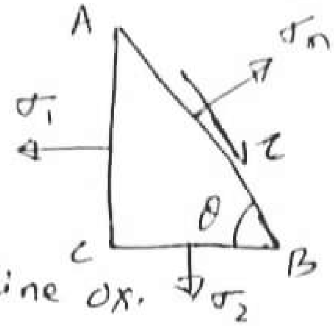
→ Now with C as centre and radius equal to CO & or CJ draw a circle. It is Known as Mohr's circle for stresses.

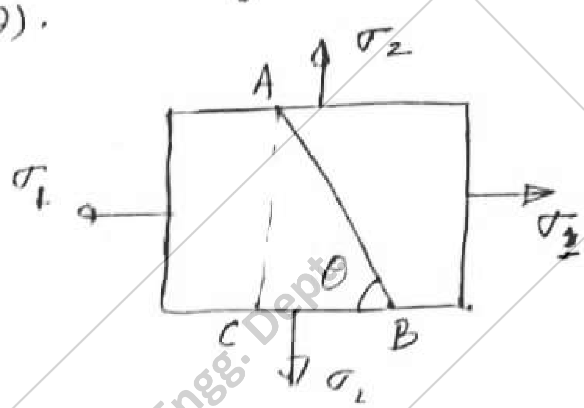
→ Through C draw a line CP making an angle of 2θ with CO in the clockwise direction meeting the circle at P. The point P represents the section AB.

→ Through P, draw PQ perpendicular to OX. Join OP.

→ Now OQ, QP & OP will give the normal, shear and resultant stress respectively to scale. The angle POJ is called the angle of obliquity (θ).

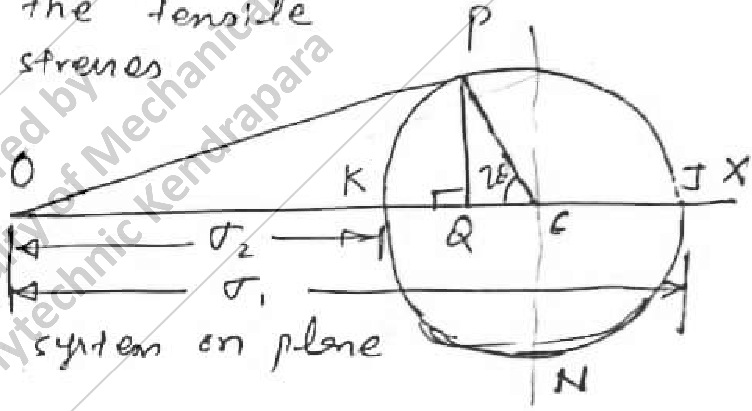
Case-2 :





→ Take some suitable point O & draw a horizontal line OX.

→ Cut off OJ & OK equal to the tensile stress (because both the senses stresses are tensile). The point J represents the stress system on plane AC and the point K represents the stress system on plane BC. Bisect JK at C.



→ Now with C as centre and radius equal to CJ to CJ draw a circle. It is known as Mohr's circle of stress.

→ Now through C, draw a line CP making an angle of 2θ with CK in clockwise direction meeting the circle at P. The point P represents the stress systems on section AB.

→ Through P draw PQ perpendicular to the line OX. Join OP.

→ Now OQ, QP and OP will give the normal, shear and resultant stress respectively to scale. Similarly CM or CN will give the maximum shear stress to the scale. The angle POC is called the angle of obliquity.

CHAPTER-04: BENDING MOMENT AND SHEAR FORCE

SOM

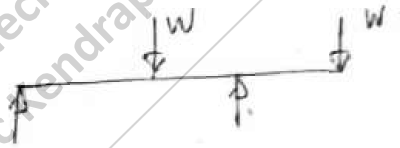
Beam:- A beam is a structural member subjected to a system of external forces at right angles to its axis.

Types of Beams:

a) Cantilever Beam → if such a member is fixed or built in at one end while its other end is free.

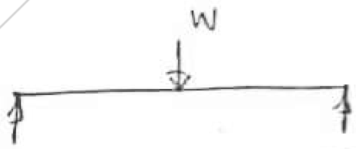
b) Simply supported Beam → If the ends of a beam are made to freely rest on supports the beam.

c) Overhanging Beam →



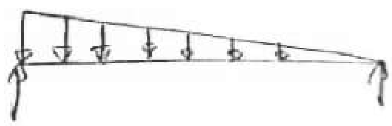
Types of Load:-

i) Concentrated or point load →



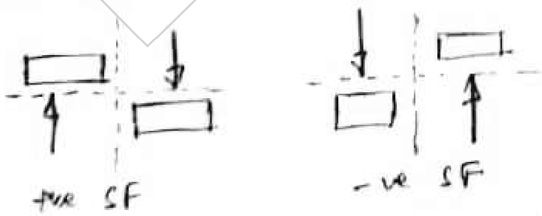
ii) Uniformly Distributed Load →

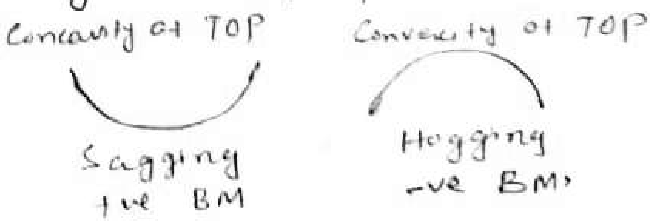
iii) Uniformly Varying Load →



Shear Force:-

→ The shear force at the cross-section of a beam, is defined as the resultant ~~unbalanced~~ vertical force to the right or left of the section.





Bending Moment:-

The bending moment at the cross-section of a beam, is defined as the algebric sum of the moments of the forces to the right or left of the section.

Take bending moment at a section as positive, when it is acting in clockwise direction to the left or in anticlockwise direction

to the right
On the other hand we take the bending moment at a section as negative, when it is acting in anticlockwise direction to the left or in clockwise direction to the right.

Response of SF & BM to different loading conditions :-

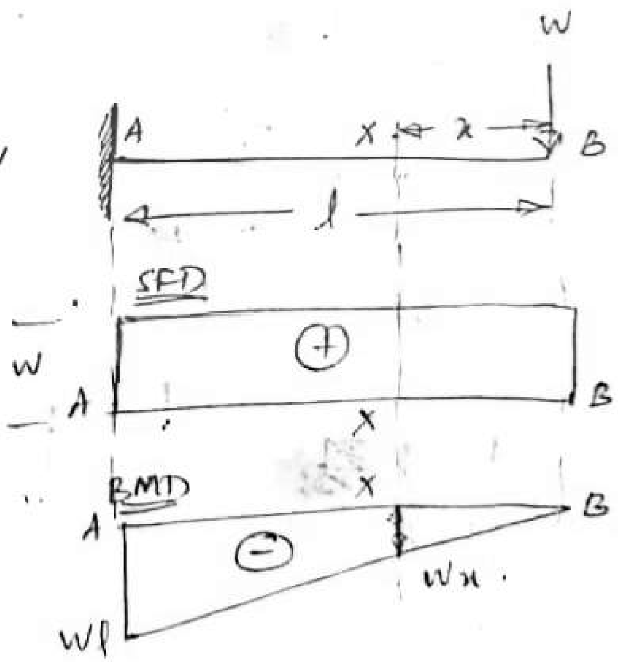
→ If there is a point load at a section on the beam, then the shear force suddenly changes but BM remains the same.

→ If there is no load between two points, then the SF doesn't change but BM changes linearly.

→ If there is UDL between two points, then the SF changes linearly but BM changes according to the parabolic law.

→ If there is UVL between two points then the SF changes according to parabolic law but BM changes according to the cubic law.

Cantilever with Point Load :-



Consider a Cantilever Beam AB of length l and carrying a point load W at its free end B.

We have the SF at any section X at a distance x from the free end,

$F_x = +W$ (+ve due to right downwards)

BM at this section,

$M_x = -Wx$ (−ve sign due to hogging i.e. clockwise rightwards)

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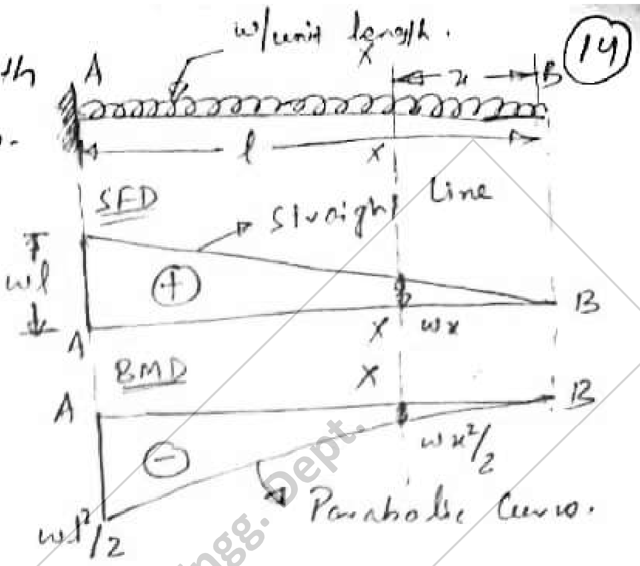
Cantilever with UDL

b) Consider a cantilever AB of length l and carrying a uniformly distributed load (UDL) of w per unit length, over the entire length of the cantilever.



We have the SF at any section X, at a distance x from B,

$$F_x = +wx$$

& BM at X,

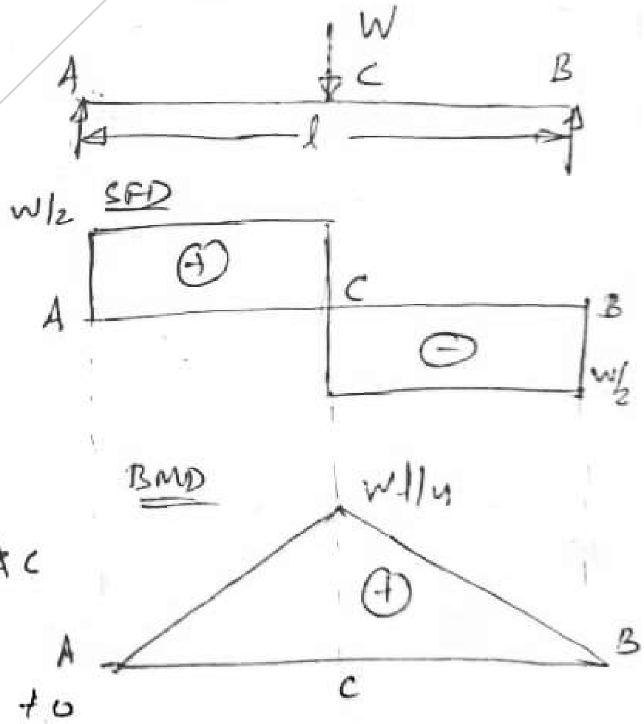
$$M_x = -wx \cdot x/2 = -\frac{wx^2}{2}$$

Simply Supported Beam with a Point Load:-

Consider a simply supported beam AB of span l and carrying a load W at its mid-point C.

Since the load is acting at the mid-point of the beam, reaction at the support A,

$$R_A = R_B = 0.5W = W/2$$



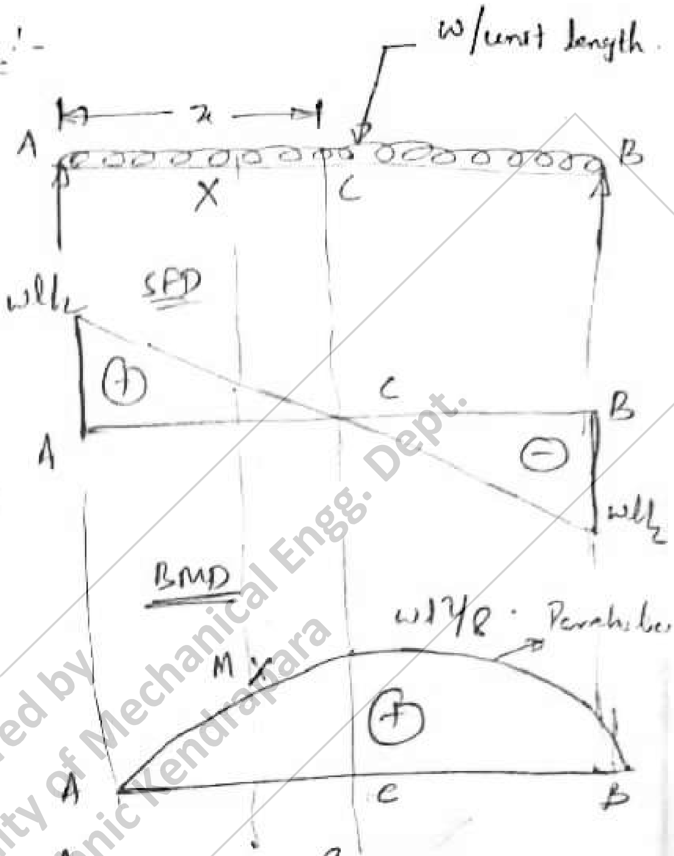
The SF at any section between A & C (i.e. upto the point just before the load W) is constant & is equal to the unbalanced vertical force $+W/2$. SF at any section between C & B (i.e. just after the load W) is also constant and is equal to the unbalanced force $-W/2$.

BM at C,

$$M_c = W/2 \times l/2 = Wl/4$$

Simply Supported Beam with UDL :-

Consider a simply supported beam AB of length l and carrying a UDL of w per unit length. So the support reaction,



$$R_A = R_B = \frac{wl}{2}$$

We have SF at any section X at a distance x from A,

$$F_X = R_A - wx = 0.5wl - wx$$

We have BM at any section at a distance x from A,

$$M_X = R_A x - \frac{wx^2}{2} = \frac{wl}{2}x - \frac{wx^2}{2}$$

BM at C at mid-point i.e. $x = l/2$,

$$M_C = \frac{wl}{2}\left(\frac{l}{2}\right) - \frac{w}{2}\left(\frac{l}{2}\right)^2 = \frac{wl^2}{8}$$

Overhanging beam :- It is a simply supported beam which overhangs (i.e. extends in the form of a cantilever) from its support. So for the purpose of SFD & BMD the overhanging beam is analysed as a combination of a simply supported beam & a cantilever.

Point of contraflexure :- As overhanging beams are analysed as a combination of simply supported & cantilever beam, and the BM in a cantilever beam is -ve & in a simply supported beam is +ve, there will be a point, where the bending moment will change sign from -ve to +ve or viceversa. Such a point where the BM changes its sign is known as point of contraflexure.

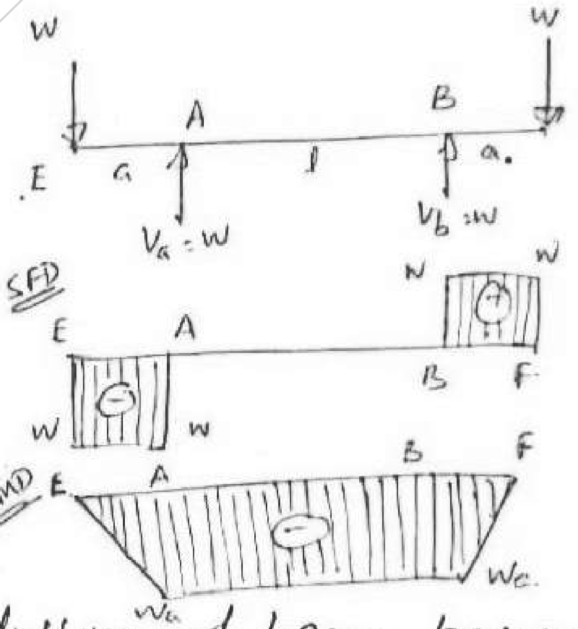
Chapter-05:

Theory of Simple Bending:

Assumptions in the theory of bending:

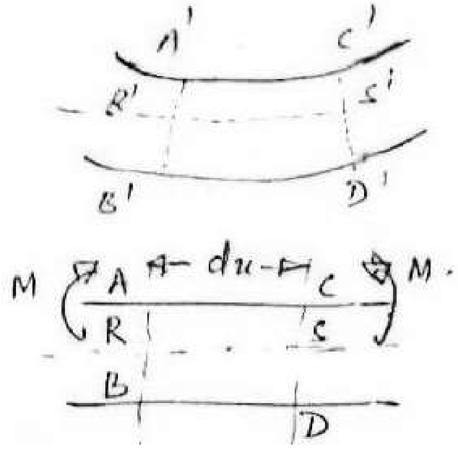
- → The material of the beam is perfectly homogeneous & isotropic.
- → The beam material is stressed within its elastic limit & thus obeys Hooke's Law.
- → The transverse sections, which were plane before bending, remain plane after bending also.
- → Each layer of the beam is free to expand or contract, independently, of the layer above or below it.
- → The value of E is the same in tension & compression.
- → The beam is in equilibrium.

Pure Bending:



Let a point load W be applied at each end of the beam. Between A & B the BM is constant & there is no SF at all between A & B i.e. between A & B the beam is absolutely free from shear but is subjected to BM Wa. This condition of beam between A & B is called pure bending or simple bending.

Theory of Simple Bending:

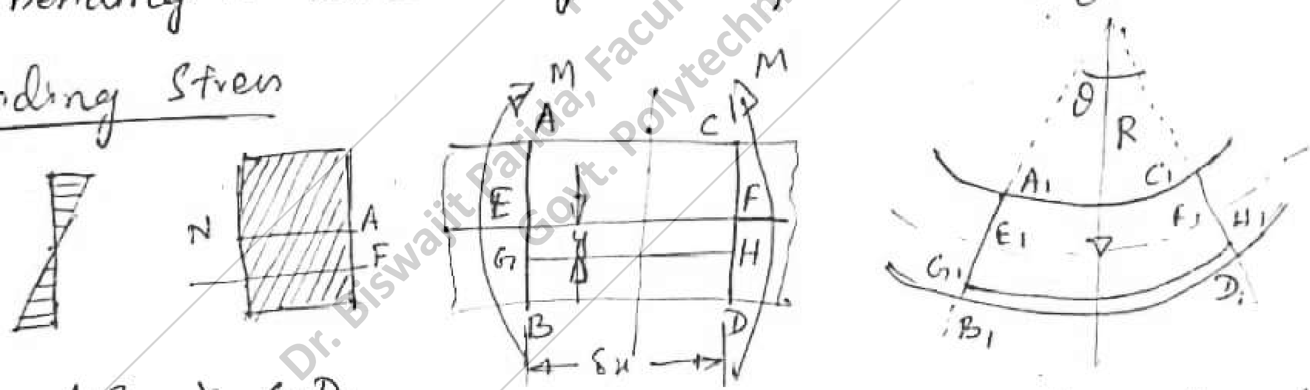


Consider a small length of a simply supported beam subjected to BM. Now consider two sections AB & CD, which are normal to the axis of the beam RS.

Consider a small length "dx" of the beam. Due to bending the top layer of the beam has suffered compression & reduced to A'C'. As we proceed towards the lower layers of the beam, ~~we find~~ the layers have no doubt suffered compression, but to less degree. The amount of extension increases as we proceed lower, until we come across the lowermost layer BD which has been stretched to B'D'.

The amount by which the layer is compressed or stretched, depends upon the position of the layer with reference to RS. The layer RS, which is neither compressed nor stretched, is known as neutral plane or neutral layer. This theory of bending is called theory of simple bending.

Bending Stress



Let A_1B_1 & C_1D_1 meet at O & angle between them is θ. Let the radius of the neutral surface be R. Consider a fibre GH at a distance y from the neutral layer.

Original length of this fibre $GH = \delta x$

After deformation of this fibre, length $= (R+y)\theta$

$\therefore EF = E_1F_1 = \delta x \Rightarrow \delta x = R\theta$

So the change in length of $GH = G_1H_1 - GH$

$= (R+y)\theta - \delta x = (R+y)\theta - R\theta = y\theta$

Strain in fibre GH,

$$= e = \frac{\text{Change in length}}{\text{Original length}} = \frac{y\theta}{R\theta} \Rightarrow e = \frac{y}{R}$$

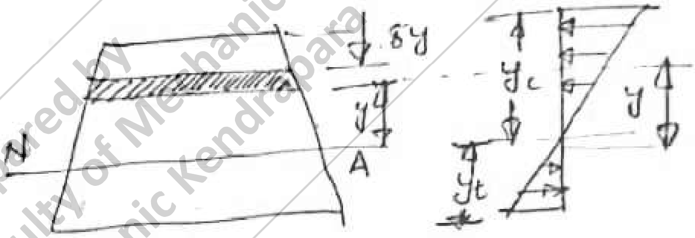
For a stress intensity σ_b the strain in the fibre,

$$\varepsilon = \frac{\sigma_b}{E}$$

$$\therefore \varepsilon = \frac{\sigma_b}{E} = \frac{y}{R} \Rightarrow \frac{\sigma_b}{y} = \frac{E}{R} \Rightarrow \boxed{\sigma_b = \frac{E}{R} y.}$$

Stress determined from the above fixture formula are called flexural stresses or bending stresses. This stress intensity in any fibre is proportional to the distance of the fibre from the neutral layer.

Neutral Axis:-



The neutral axis of any transverse section of a beam is the line of intersection of the neutral layer with the transverse section.

Let R be the radius of curvature of the neutral layer. So the stress at any point distance y from the NA is given by,

$$\sigma_b = \frac{E}{R} \times y.$$

Consider an elemental area δa at a distance y from NA.

Stress on elemental area $= \sigma_b = \frac{E}{R} y.$

$\therefore$ Thrust on the elemental area $= \sigma_b \, \delta a = \frac{E}{R} y \cdot \delta a.$

$\therefore$ Total thrust on the beam section $= \frac{E}{R} \sum_{y_t}^{y_c} y \, \delta a.$

Since no axial load has been applied, the total thrust on the beam section equals zero.

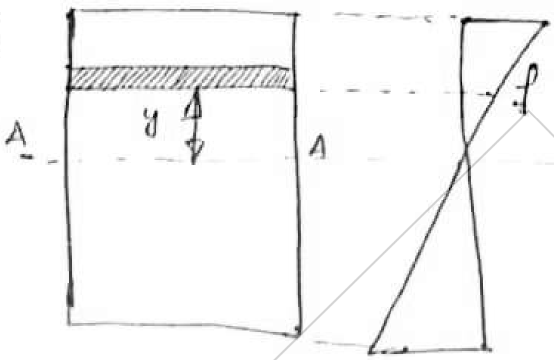
$$\frac{E}{R} \sum_{y_t}^{y_c} y \, \delta a = 0 \Rightarrow \sum_{y_t}^{y_c} y \, \delta a = 0$$

This is possible only when the NA is a centroidal axis.

Moment of Resistance:-

Consider an elemental area δa at a distance y from the NA

The stress intensity on the elemental area = $\sigma_b = \frac{E}{R} y$



Thrust on the elemental area.

$= \sigma_b \, \delta a = \frac{E}{R} y \, \delta a.$

MR offered by the elemental area

= moment of the thrust about NA $= \frac{E}{R} y^2 \delta a.$

∴ Total MR offered by the beam section $= M = \frac{E}{R} \sum_{y_t}^{y_c} y^2 \delta a.$

But $\sum_{y_t}^{y_c} y^2 \delta a$ is the MI of the beam section about the NA $\Rightarrow (I)$

$\therefore \quad M = \frac{E}{R} I \Rightarrow \frac{M}{I} = \frac{E}{R}$

$$\Rightarrow \boxed{\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}} \qquad \left(\because \frac{\sigma_b}{y} = \frac{E}{R} \right)$$

Section Modulus :-

Let M be the MR of a beam section & I the MI of the section about the NA.

So the stress at any point of the section at a distance y from the NA is given by,

$$\sigma_b = \frac{M}{I} y.$$

If y_{max} is the most distant point of section from the NA we have,

$$\sigma_{b\,max} = \frac{M}{I} y_{max}.$$

$$\Rightarrow M = \sigma_{b\,max} \cdot \frac{I}{y_{max}} = \sigma_{b\,max} \times Z$$

where, $Z = \frac{I}{y_{max}} = \frac{\text{MI about the NA}}{\text{Distance of most distant point of the section from NA}}$

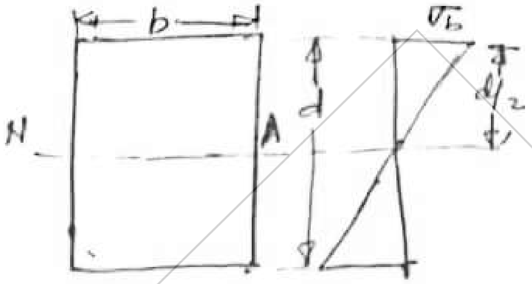
= Section modulus.

Hence for a beam of a given material the greatest MR of the beam section can offer is, $M = \sigma_{b\,safe} \cdot Z$

where $\sigma_{b\,safe} \rightarrow$ permissible bending stress.

Section Modulus for various shapes of beam section :-

i) Rectangular Section



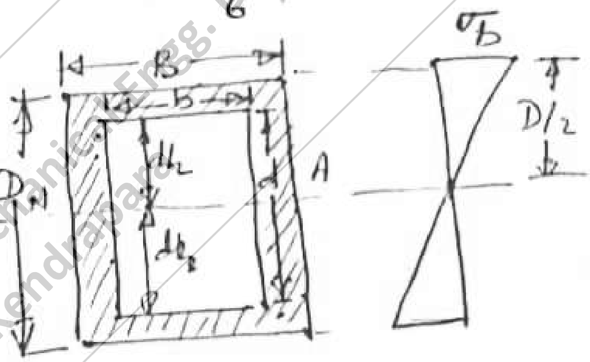
$I = \frac{bd^3}{12}$ & $y_{max} = d/2$

$Z = \frac{bd^3}{12} \times \frac{2}{d} = \frac{bd^2}{6}$

Moment of Resistance $= M = \sigma_b Z = \sigma_b \frac{bd^2}{6}$

$\Rightarrow \boxed{M = \frac{1}{6} \sigma_b bd^2}$

ii) Hollow rectangular section :-



$I = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{1}{12}[BD^3 - bd^3]$

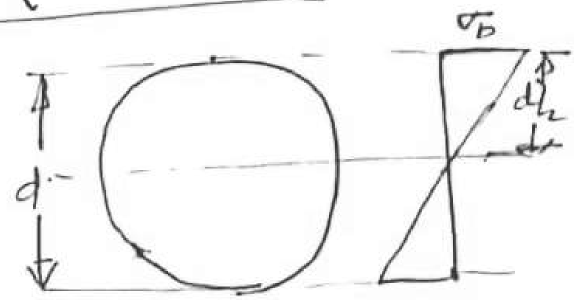
$y_{max} = D/2$

Section Modulus, $Z = \frac{I}{y_{max}} = \frac{1}{12}[BD^3 - bd^3] \; 2/D$

$\Rightarrow Z = \frac{BD^3 - bd^3}{6D}$

$MR = M = \sigma_b Z \Rightarrow \boxed{M = \frac{1}{6} \sigma_b \left(\frac{BD^3 - bd^3}{D}\right)}$

iii) Circular section

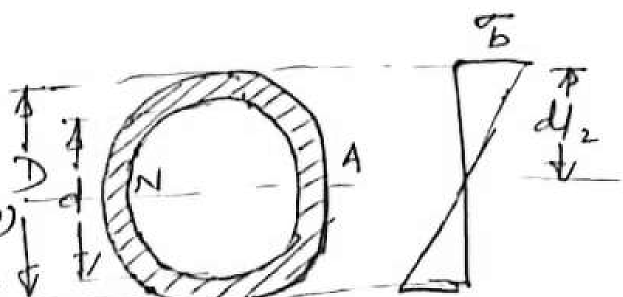


$I = \frac{\pi d^4}{64}$, $y_{max} = d/2$

Section modulus $= Z = \frac{I}{y_{max}} = \frac{\pi d^3}{32}$

Moment of resistance $= M = \sigma_b Z \Rightarrow \boxed{M = \sigma_b \frac{\pi d^3}{32}}$

iv) Hollow Circular Section



$I = \frac{\pi}{64}[D^4 - d^4]$, $y_{max} = D/2$

Section modulus $= Z = \frac{I}{y_{max}} = \frac{\pi (D^4 - d^4)}{32D}$

Moment of resistance $= \boxed{M = \sigma_b \frac{\pi (D^4 - d^4)}{32D}}$

Chapter: 06

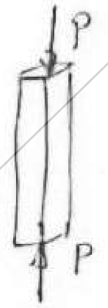
Combined Direct & Bending Stresses

SOM

Column:- are vertical members used in building frames and called vertical struts.

Strut:- A structural member, subjected to an axial compressive force, is called strut.

Axial Load:-



Consider a short column of uniform cross-sectional area A, subjected to an axial load P (compressive), by which a compressive or crushing stress has been developed,

$$\sigma_c = \frac{P_c}{A} \Rightarrow P_c = \sigma_c A \quad \text{(Crushing Load)}$$

Buckling Load / Crippling Load / Critical Load:-

The columns are considerably long in comparison with their lateral dimensions. So these members start bending, i.e. buckling when the axial load reaches a critical value. This load at which the member just buckles is called the buckling load or critical load or crippling load. The buckling load is less than the crushing load.

Euler's theory of long columns:-

Mr. Euler derived an equation to study the stability of long columns for buckling load based on bending stress. Here the effect of direct stress is neglected which may be justified with the statement that the direct stress induced in a long column is negligible as compared to the bending stress.

Effective Length of a Column:-

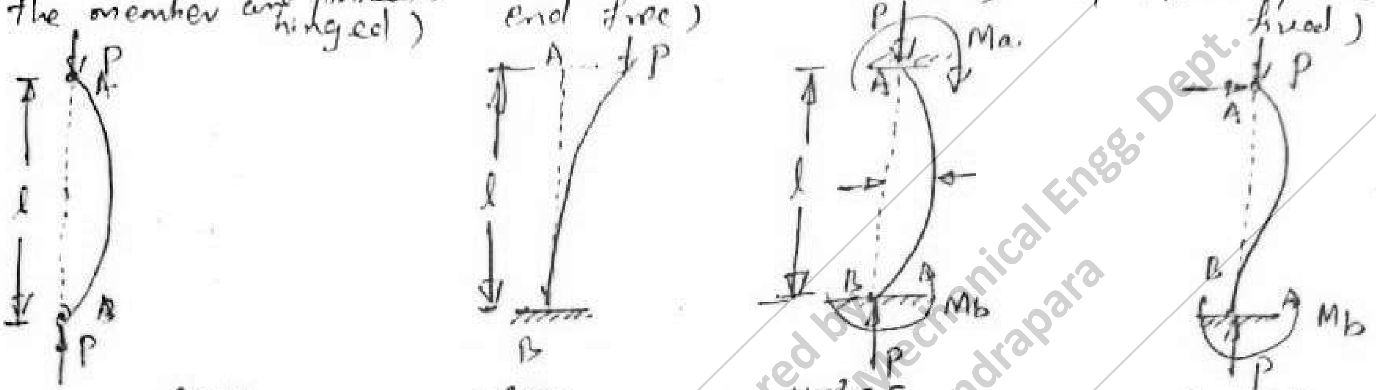
The effective length of a given column with given end conditions is the length of an equivalent column of the same material & section with hinged ends having the value of the crippling load equal to that of the given column.

Case-1 (When both ends of the member are pinned/hinged)

Case-2 (One end fixed & other end free)

Case-3 (Both ends are fixed)

Case-4 (One end fixed & other is pinned/hinged)



Case-1: $P = \frac{\pi^2 E I}{l^2}$, $l = L$

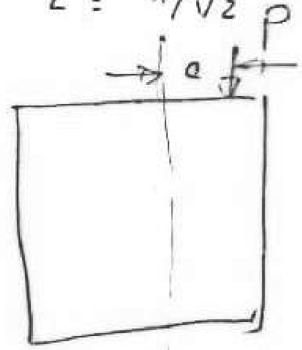
Case-2: $P = \frac{\pi^2 E I}{4 l^2}$, $L = 2l$

Case-3: $P = \frac{4\pi^2 E I}{l^2}$, $L = l/2$

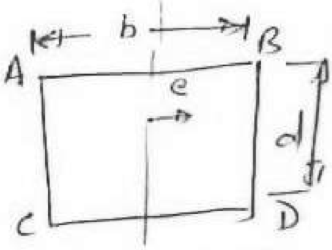
Case-4: $P = \frac{2\pi^2 E I}{l^2}$, $L = l/\sqrt{2}$

Eccentric load in columns:

A load whose line of action doesn't coincide with the axis of a column is called eccentric load.



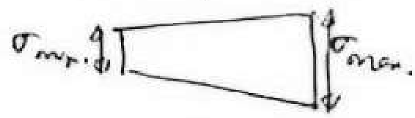
Consider a column ABCD subjected to an eccentric load P about y-axis at a distance e.



Area of the section = $b \times d$.

b & d → width & thickness of the column.

M.I, $I = (db^3)/12$



Section Modulus, $Z = \frac{I}{y} = \frac{db^3/12}{b/2} = \frac{db^2}{6}$

Direct stress, $\sigma_0 = P/A$

Moment due to load, $M = P \times e$

Bending stress at any point of column section at a distance y from y-axis, $\sigma_b = (M/I)\,y = M/Z$

at $y = b/2$ → $\sigma_b = \frac{M \times b/2}{db^3/12} = \frac{6 P \times e}{A \times b}$

Total Stress = Direct Stress + Bending stress $= \frac{P}{A} \pm \frac{M}{Z} = \frac{P}{A} \pm \frac{6Pe}{Ab}$

+ → Max^m

− → Min^m

Chapter :07

TORSION

SOM

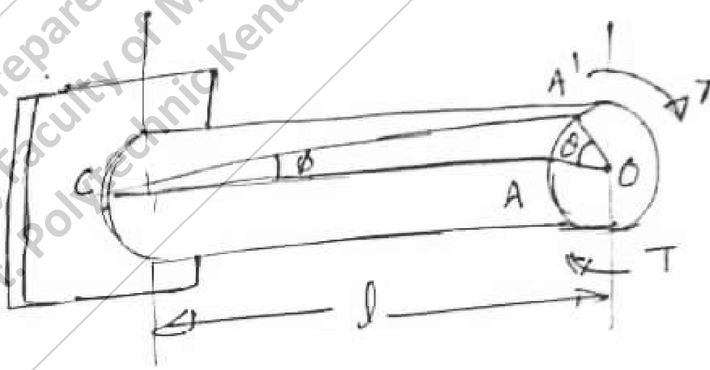
Pure Torsion:-

→ A shaft of circular section is said to be in pure torsion when it is subjected to equal & opposite end couples whose axes coincide with the axis of the shaft.

→ While a beam bends as an effect of BM, a shaft twists as an effect of torsion.

→ At any point in the section of the shaft, a shear stress is induced or more exactly, the state of stress at any point in the cross-section of the shaft is one of pure shear.

Theory of Pure Torsion:



Consider a circular shaft fixed at one end and subjected to a torque at the other end.

Let T → Torque in N-mm.

l → length of the shaft in mm.

R → radius of the circular shaft in mm.

Due to the application of the twisting moment (T) the shaft will be subjected to shear stresses as the line CA on the surface of the shaft is deformed from CA to CA' & OA to OA'.

Then angle ∠ACA' = φ represents the shear strain of the shaft material of the surface.

For very small values of φ we can write,

$$AA' = \phi l \Rightarrow \phi = \frac{AA'}{l}$$

we have, $AA' = R\theta$

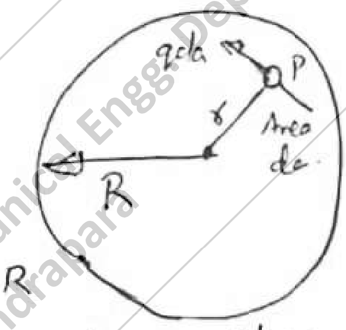
$$\Rightarrow \phi = \frac{R\theta}{l}$$

If τ is the shear stress intensity on the outermost layer & C is the modulus of rigidity of the shaft,

$$\phi = \frac{\tau}{C} \Rightarrow \frac{\tau}{C} = \frac{R\theta}{l} \Rightarrow \frac{\tau}{R} = \frac{C\theta}{l}$$

If q is the stress intensity at any radial distance r from the centre of the shaft,

$$\boxed{\frac{q}{r} = \frac{\tau}{R} = \frac{C\theta}{l}}$$



Torsional Moment of Resistance:-

Consider a section of the shaft of radius R subjected to pure torsion, and τ be the maximum shear stress occurring at the surface.

Consider an elemental area da at a distance r from the axis of the shaft.

Shear stress offered by the elemental area $= q = \frac{r}{R}\tau$.

Shear resistance " " " " $= q \cdot da = \left(\frac{r}{R}\tau\right) da$.

∴ Moment of resistance " " " " $= \left(\frac{r}{R}\tau \cdot da\right) r$

∴ Total " " " " $= T = \frac{\tau}{R} \Sigma da \cdot r^2$

$\Sigma da \cdot r^2$ is the MI of the shaft section about the axis of the shaft $= I_p$ (Polar MI)

$$\Rightarrow T = \frac{\tau}{R} \cdot I_p \Rightarrow \frac{T}{I_p} = \frac{\tau}{R}$$

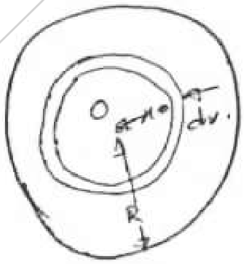
$$\Rightarrow \boxed{\frac{T}{I_p} = \frac{\tau}{R} = \frac{C\theta}{l}} \longrightarrow \text{Torsion equation.}$$

SOM

Assumptions in the theory of pure torsion :-

- The material of the shaft is uniform throughout.
- The twist along the shaft is uniform.
- The shaft is of uniform circular section throughout.
- Cross-sections of the shaft, which are plane before twist remain plane after twist.
- All radii which are straight before twist remain straight after twist.

Torsion equation for a solid circular shaft :-



Consider a shaft subjected to a torque T. Let us consider an element of area da of thickness dx at a distance x from the centre of the shaft.

$$\therefore da = 2\pi x \cdot dx.$$

& shear stress at this section,

$$\tau_x = \tau \times \frac{x}{R} \quad , \tau \to \text{Maximum shear stress.}$$

$\therefore$ Turning force = Shear stress $\times$ Area.

$$= \tau_x \times da = \tau \times \frac{x}{R} \times da.$$

$$= \tau \frac{x}{R} \times 2\pi x \cdot dx = \frac{2\pi\tau}{R} \cdot x^2 dx,$$

We have the turning moment of this element,

dT = Turning force $\times$ Distance of element from shaft axis.

$$= \frac{2\pi\tau}{R} \cdot x^2 dx \cdot x = \frac{2\pi\tau}{R} x^3 \cdot dx.$$

$\therefore$ The total torque, which the shaft can withstand, may be found out by integrating the above eqn,

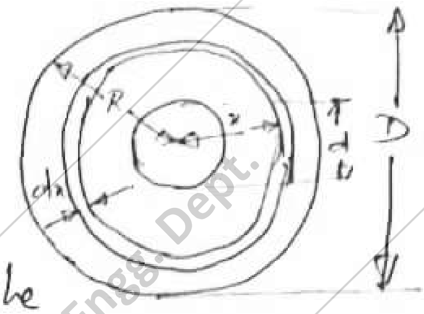
$$T = \int_0^R \frac{2\pi\tau}{R} \cdot x^3 \cdot dx = \frac{2\pi\tau}{R} \int_0^R x^2 \cdot dx$$

$$= \frac{2\pi\tau}{R} \left[\frac{x^4}{4}\right]_0^R = \frac{\pi}{16} \times \tau \times D^3 \text{ N-mm.}$$

$$\Rightarrow T = \frac{\pi}{16} \times \tau \times D^3$$

Strength of a Hollow shaft / Torsion Eqⁿ for a hollow shaft :-

Consider a hollow circular shaft subjected to some torque T.



Let $R \rightarrow$ outer radius of shaft in mm.

$r \rightarrow$ Inner " "

$\tau \rightarrow$ Maxm. shear stress developed in the outermost layer of the shaft material.

Now consider an elementary ring of thickness dx at a distance x from the centre.

we have area of the ring, $da = 2\pi x \cdot dx$.

$\therefore$ shear stress at this section,

$$\tau_x = \tau \times \frac{x}{R}$$

$\therefore$ Turning force = Stress × Area = $\tau_x \times da$

$$= \tau \times \frac{x}{R} \times 2\pi x\, dx = \frac{2\pi\tau}{R} x^2 \cdot dx.$$

We have the turning moment of this element,

dT = Turning force × Distance of element from axis of the shaft.

$$= \frac{2\pi\tau}{R} x^2 \cdot dx \cdot x = \frac{2\pi\tau}{R} x^3 \cdot dx.$$

The total torque, which the shaft can transmit, may be found out by integrating the above equation.

$$T = \int_r^R \frac{2\pi\tau}{R} x^3 \cdot dx = \frac{2\pi\tau}{R} \int_r^R x^3 \cdot dx.$$

$$= \frac{2\pi\tau}{R} \left[\frac{x^4}{4}\right]_r^R = \frac{2\pi\tau}{R} \left(\frac{R^4 - r^4}{4}\right)$$

$$\Rightarrow T = \frac{\pi}{16} \times \tau \times \left(\frac{D^4 - d^4}{D}\right)$$

Power Transmitted by a shaft :-

Let N → No. of revolutions per minute.

T → Average torque in kN-m.

Workdone per minute = Force × Distance = $T \times 2\pi N = 2\pi NT$

Workdone per second = $\frac{2\pi NT}{60}$ kN-m. = Power transmitted.

$$\Rightarrow \boxed{P = \frac{2\pi NT}{60} \text{ kW}}$$