Lecture Notes

on

Fluid Mechanics

(Th.3)

4^{th} Semester, Mechanical Engg.

Prepared by

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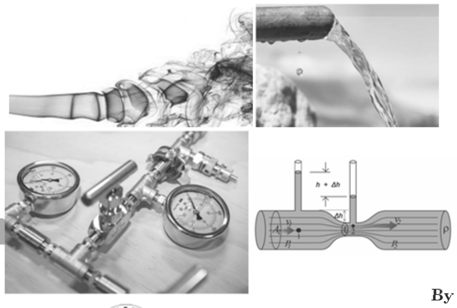
FLUID MECHANICS



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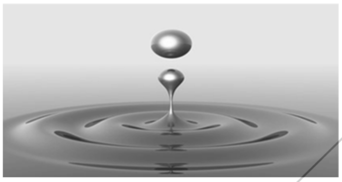

INTRODUCTION

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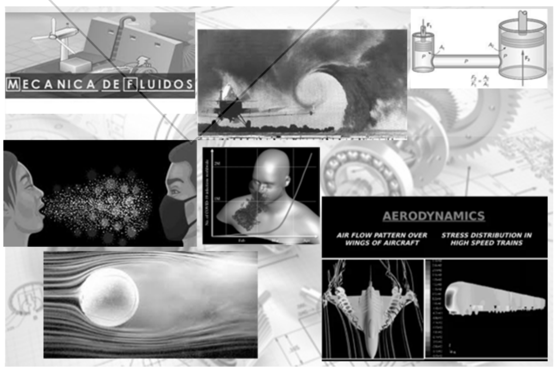
WHY TO STUDY FLUID MECHANICS....?



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Why To Study Fluid Mechanics....?

Common Applications of Fluids

- Hydroelectric Power Plants
- Hydraulic machines
- Refrigerators and Air Conditioners
- Thermal Power Plants
- Nuclear power plants
- Fluids as a Renewable Energy Source
- Operating Various Instruments
- Heat Engines

Thank You

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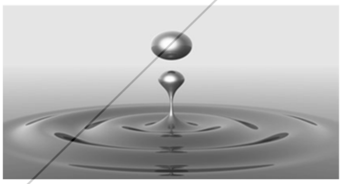

Introduction

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What We Are Going To Cover In Fluid Mechanics….?

1.0 Properties of Fluid
- 1.1 Define fluid
- 1.2 Description of fluid properties like Density, Specific weight, specific gravity, specific volume and solve simple problems.
- 1.3 Definitions and Units of Dynamic viscosity, kinematic viscosity, surface tension Capillary phenomenon

2.0 Fluid Pressure and its measurements
- 2.1 Definitions and units of fluid pressure, pressure intensity and pressure head.
- 2.2 Statement of Pascal's Law.
- 2.3 Concept of atmospheric pressure, gauge pressure, vacuum pressure and absolute pressure
- 2.4 Pressure measuring instruments
  Manometers (Simple and Differential)
  - 2.4.1 Bourdon tube pressure gauge(Simple Numerical)
- 2.5 Solve simple problems on Manometer.

3.0 Hydrostatics
- 3.1 Definition of hydrostatic pressure
- 3.2 Total pressure and centre of pressure on immersed bodies(Horizontal and Vertical Bodies)
- 3.3 Solve Simple problems.
- 3.4 Archimedes 'principle, concept of buoyancy, meta center and meta centric height (Definition only)
- 3.5 Concept of floatation

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What We Are Going To Cover In Fluid Mechanics….?

4.0 Kinematics of Flow
- 4.1 Types of fluid flow
- 4.2 Continuity equation(Statement and proof for one dimensional flow)
- 4.3 Bernoulli's theorem(Statement and proof)
  Applications and limitations of Bernoulli's theorem (Venturimeter, pitot tube)
- 4.4 Solve simple problems

5.0 Orifices, notches & weirs
- 5.1 Define orifice
- 5.2 Flow through orifice
- 5.3Orifices coefficient & the relation between the orifice coefficients
- 5.4 Classifications of notches & weirs
- 5.5 Discharge over a rectangular notch or weir
- 5.6 Discharge over a triangular notch or weir
- 5.7 Simple problems on above

6.0 Flow through pipe
- 6.1 Definition of pipe.
- 6.2 Loss of energy in pipes.
- 6.3 Head loss due to friction: Darcy's and Chezy's formula (Expression only)
- 6.4 Solve Problems using Darcy's and Chezy's formula.
- 6.5 Hydraulic gradient and total gradient line

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What We Are Going To Cover In Fluid Mechanics….?

7.0 Impact of jets
- 7.1 Impact of jet on fixed and moving vertical flat plates
- 7.2 Derivation of work done on series of vanes and condition for maximum efficiency.
- 7.3 Impact of jet on moving curved vanes, illustration using velocity triangles, derivation of work done, efficiency.

Learning Resources:

Sl No.	Name of the Book	Author Name	Publisher
1.	Text Book of Fluid Mechanics	R.K.Bansal	Laxmi
2.	Text Book of Fluid Mechanics	R.S khurmi	S.Chand
3.	Text Book of Fluid Mechanics	R.K.Rajput	S.Chand
4.	Text Book of Fluid Mechanics	Modi & Seth	Rajson's pub. Pvt. lt

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CHAPTER-01:
PROPERTIES OF FLUID

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Introduction

Fluid

In very simple way we can say that **"Which Can Flow"**

Fluid Mechanics

→ is that branch of science which deals with the behaviour of the fluids at rest as well as in motion

→ Study of fluids at rest is called fluid statics

→ study of fluids in motion without considering pressure forces is called fluid Kinamatics and with pressure forces is called fluid dynamics.

Properties of Fluid

1) **Density** :- is defined as the ratio of the mass of a fluid to its volume.

$$\rho = \frac{\text{Mass of the fluid}}{\text{Volume of the fluid}} \ kg/m^3 \Rightarrow \rho = \frac{m}{V}$$

$$\rho_{water} = 1 \ gm/cm^3 = 1000 \ kg/m^3$$

2) **Specific wt. or Wt. density** :- is the ratio between wt. of the fluid to its volume.

$$w = \frac{\text{wt. of fluid}}{\text{Vol. of fluid}} = \frac{\text{Mass} \times \text{Accl}^n \text{ due to gravity}}{\text{Vol. of fluid}}$$

$$= \frac{\text{Mass}}{\text{Vol.}} \times g = \rho g.$$

$$\Rightarrow w = \rho g$$

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Properties of Fluid

3) **Specific vol.** :- is the vol. of a fluid occupied by a unit mass.

$$\text{Specific vol.} = \frac{\text{Vol. of fluid}}{\text{Mass of fluid}} = \frac{1}{\text{Mass / vol.}} = \frac{1}{\rho} = m^3/kg.$$

4) **Specific Gravity** :- is defined as the ratio of density or wt. density of a fluid to the density of a standard fluid.

→ also called relative density

→ For liquids, standard fluid is water
gas, standard fluid is air

→ For liquids, standard fluid is water
gas, standard fluid is ~~gas~~ air.

$$S_{liquid} = \frac{\rho_{liquid}}{\rho_{water}} \quad \& \quad S_{gas} = \frac{\rho_{gas}}{\rho_{air}}$$

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Properties of Fluid

$$= \frac{\text{wt. density}_{liquid}}{\text{wt. density}_{water}} \quad \& \quad S_{gas} = \frac{\text{wt. density}_{liquid}}{\text{wt. density}_{water}}$$

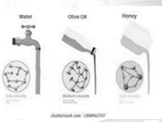
Density of liquid = S × Density of water

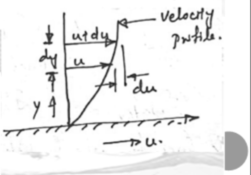
" mercury = 13.6 × 1000 = 13600 kg/m^3.

Viscosity :- is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of fluid.

→ Shear stress is proportional to rate of change of velocity w.r.t. y.

$$\tau \propto \frac{du}{dy} \Rightarrow \tau = \mu \frac{du}{dy}.$$





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Properties of Fluid

$\mu \rightarrow$ Const. of proportionality / Co-efficient of dynamic viscosity / Viscosity.

$\frac{du}{dy} \rightarrow$ rate of shear strain / rate of shear deformation / velocity gradient

$\rightarrow \mu = \frac{\tau}{(du/dy)}$ (defined as the shear stress required to produce unit rate of shear strain)

$$\mu = \frac{\text{Shear stress}}{\left(\frac{\text{Change in velocity}}{\text{Change in distance}}\right)} = \frac{\text{Force / Area}}{\left(\frac{\text{Length}}{\text{Time}}\right) \times \frac{1}{\text{Length}}} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2}$$

SI $\rightarrow \frac{N\text{-Sec.}}{m^2}$, CGS $\rightarrow \frac{\text{Dyne-Sec.}}{cm^2}$ (1 poise), MKS $\rightarrow \frac{\text{Kgf-Sec.}}{m^2}$ (1 kgf = 9.81 N)

$$1 \text{ poise} = \frac{1}{10} \frac{N\text{-s}}{m^2}$$

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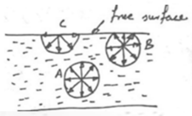
Properties of Fluid

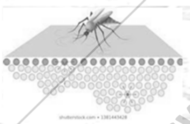
Kinematic Viscosity :- is defined as the ratio between the dynamic viscosity and density of fluid.

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \mu/\rho = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \times \frac{\text{Mass}}{(\text{Length})^3}} = \frac{(\text{Length})^2}{\text{Time}}$$

1 stoke = 1 cm^2/s https://www.youtube.com/watch?v=I8xFVGdkkUU&t=1s

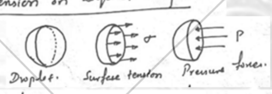
Surface Tension :- is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.





Properties of Fluid

i) **Surface Tension on liquid droplet** :-



Tensile force due to surface tension acting around the circumference of the cut portion $= \sigma \times \pi d$

Pressure force on the area $\frac{\pi}{4}d^2 = P \times \frac{\pi}{4}d^2$

For equilibrium condition,

$$P \times \frac{\pi}{4} d^2 = \sigma \times \pi d \Rightarrow \boxed{P = \frac{4\sigma}{d}}$$

ii) **Surface Tension on a hollow bubble** :

As it has two surfaces in contact with air, one inside & other outside, thus two surfaces are subjected to surface tension.

$$\Rightarrow P \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d) \Rightarrow \boxed{P = \frac{8\sigma}{d}}$$

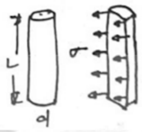
Properties of Fluid

iii) Surface Tension on a liquid jet :

Force due to pressure $= P \times (L \times d)$

due to surface tension $= \sigma \times 2L$

$\Rightarrow P \times L \times d = \sigma \times 2L \Rightarrow \boxed{P = \frac{2\sigma}{d}}$



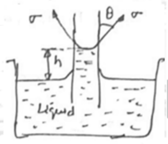
Capillarity:

https://www.youtube.com/watch?v=PP9mn-X9i2Q

→ is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid.

→ Its value depends upon the specific wt. of the liquid, diameter of the tube & surface tension of the liquid.

Expression for capillary rise.



wt. of liquid of height h in the tube.

$= V \rho g = (\text{Area of the tube} \times h) \times \rho \times g$

$= \frac{\pi}{4} \times d^2 \times h \times \rho \times g.$

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Properties of Fluid

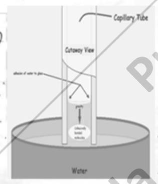
Vertical component of the surface tensile force,

$= (\sigma \times \pi d) \times \cos\theta.$

For equilibrium

$\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos\theta$

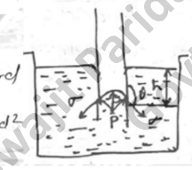
$\Rightarrow \boxed{h = \frac{4\sigma \cos\theta}{\rho g d}}$



For water & clean glass tube, $\theta \approx 0 \Rightarrow \cos\theta \approx 1$

$\Rightarrow h = 4\sigma / \rho g d$

ii) Expression for capillary fall :-



Surface tension force acting in the downward direction $= \sigma \times \pi d \times \cos\theta.$

Hydrostatic force acting upward $= \rho g h \times \frac{\pi}{4} d^2$

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Properties of Fluid

$\Rightarrow \sigma \times \pi d \times \cos\theta = \rho g h \times \frac{\pi}{4} d^2$

$\Rightarrow \boxed{h = \frac{4\sigma \cos\theta}{\rho g d}}$ $\theta_{\text{mercury \& glass tube}} = 128°$

→ is expressed in terms of cm or mm of liquid

Surface Tension:

$\sigma = F/L$ N/m, Dyne/cm, Kgf/cm

The magnitude of this tensile force per unit length of the free surface will have the same value as the surface energy per unit area.

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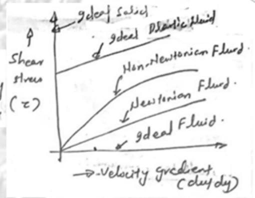
Properties of Fluid

Newton's law of viscosity

→ It states that the shear stress on a fluid element layer is directly proportional to the rate of shear strain.



$$\tau \propto \frac{du}{dy}$$

$$= \mu \frac{du}{dy}$$

→ Fluids which obey the above eq^n or law are known as Newtonian fluids & the fluids which do not obey the law are called Non-Newtonian fluids.

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REVISING

CHAPTER-01:

PROPERTIES OF FLUID

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Class-15

CHAPTER-02:

FLUID PRESSURE AND ITS MEASUREMENTS

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Fluid Pressure

Fluid Pressure:-

When a fluid is contained in a vessel, it exerts force at all points on the sides & bottoms of the container, and this force per unit area is called pressure.

$$P = F/A = \underset{(1\ \text{Pascal})}{N/m^2},\ \frac{Dyne}{cm^2},\ kgf/m^2$$

Pressure Head:

→ A liquid is subjected to pressure due to its own weight, which increases as the depth of the liquid increases.

$$P = \frac{F}{A} = \frac{\text{wt. of the liquid in the cylinder}}{\text{Area of the cylinder}}$$

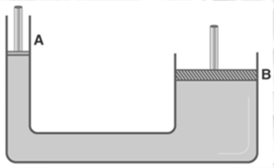
$$= \frac{\text{Sp. wt.} \times \text{Vol.}}{A} = \frac{w \times Ah}{A} \Rightarrow P = wh \Rightarrow \boxed{P = \rho gh.}$$

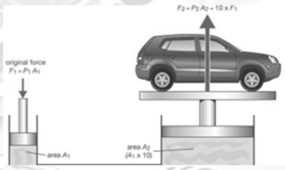
→ So pressure intensity at any point in a liquid is proportional to its depth.

Fluid Pressure (contd…)

Pascal's Law

Consider the image given below. The column is filled with water and the ends of each column A and B have been blocked by a piston. If piston A is pressed, what do you think will happen to piston B? Piston B is going to rise up. We have just applied Pascal's law to our fluid pressure.





https://www.youtube.com/watch?v=hV5IEooHqIw
https://www.youtube.com/watch?v=tAyFKf7zxFg

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions.

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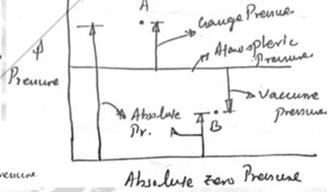
Fluid Pressure and Its Measurements

Atmospheric Pressure The atmospheric air exerts a normal pressure upon all surfaces with which it is contact & known as atmospheric pressure.



ii) Absolute Pressure:- It is defined as the pressure which is measured with reference to absolute zero pressure

iii) Gauge Pressure:. It is defined as the pressure which is measured with the help of a pressure measuring instrument in which the atmospheric pressure is taken as datum.

iv) Vacuum Pressure:- It is defined as the pressure below the atmospheric pressure.

$P_{absolute} = P_{atmospheric} + P_{gauge}$ $P_{vacuum} = P_{atmospheric} - P_{absolute}$

Fluid Pressure and Its Measurements

Pressure Measuring Instruments :-

- Manometers
  - Simple Manometers
    - Piezometer
    - U-tube
    - Single Column
      - Vertical
      - Inclined
  - Differential Manometers
    - U-tube
    - Inverted U-tube
- Mechanical Gauges
  - Diaphragm pr. gauge
  - Bourdon tube pr. gauge
  - Dead-wt. pr. gauge
  - Bellow pr. gauge

Manometers :- are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid

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Fluid Pressure and Its Measurements

1) MANOMETERS

Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid.

a) SIMPLE MANOMETERS

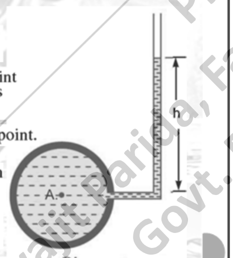
A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere.

i) Piezometer.

- It is the simplest form of manometer used for measuring gauge pressures.
- One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere
- The rise of liquid gives the pressure head at that point.

If at a point A, the height of liquid say water is h in piezometer tube, then pressure at A

$$= \rho \times g \times h \ \frac{\text{N}}{\text{m}^2}.$$



Piezometer.

https://www.youtube.com/watch?v=SR9rxSgLQEQ
https://www.youtube.com/watch?v=awJQ6eaA1cw

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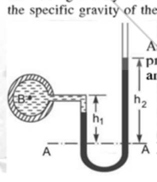
Fluid Pressure and Its Measurements

ii) U-tube Manometer

- It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere
- The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.

(*a*) For Gauge Pressure.



(a) For gauge pressure

As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum line A-A in the left column and in the right column of U-tube manometer should be same.

Pressure above A-A in the left column $= p + \rho_1 \times g \times h_1$

Pressure above A-A in the right column $= \rho_2 \times g \times h_2$

Hence equating the two pressures

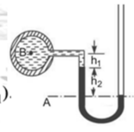
$\therefore \quad p + \rho_1 g h_1 = \rho_2 g h_2, \ p = (\rho_2 g h_2 - \rho_1 \times g \times h_1).$

(*b*) For Vacuum Pressure.

Pressure above A-A in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + p$

Pressure head in the right column above A-A $= 0$

$\therefore \quad \rho_2 g h_2 + \rho_1 g h_1 + p = 0 \quad \therefore \quad p = -(\rho_2 g h_2 + \rho_1 g h_1).$



(b) For vacuum pressure

https://www.youtube.com/watch?v=2P5_J5JEHTQ
https://www.youtube.com/watch?v=9eq7NE6OsX4
https://www.youtube.com/watch?v=aetwzRJ4CbI

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Fluid Pressure and Its Measurements

iii) Single Column Manometer. Single column manometer is a modified form of a U-tube manometer in which a reservoir, having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one of the limbs (say left limb) of the manometer

Due to large cross-sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb.

– Vertical Single Column Manometer

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

$\therefore\ A \times \Delta h = a \times h_2 \qquad \therefore\ \Delta h = \dfrac{a \times h_2}{A}$

Then pressure in the right limb above *Y-Y*.

$= \rho_2 \times g \times (\Delta h + h_2)$

Pressure in the left limb above *Y-Y*

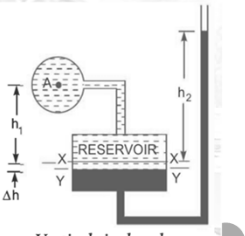
$= \rho_1 \times g \times (\Delta h + h_1) + p_A$

Equating these pressures, we have

$$\rho_2 \times g \times (\Delta h + h_2) = \rho_1 \times g \times (\Delta h + h_1) + p_A$$

$$p_A = \rho_2 g\,(\Delta h + h_2) - \rho_1 g(\Delta h + h_1)$$

$$= \Delta h[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$



Vertical single column manometer.

https://www.youtube.com/watch?v=g0kpcCBuXe0
https://www.youtube.com/watch?v=MpXhXVF9-HM

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Fluid Pressure and Its Measurements

$\Rightarrow P_A = \dfrac{a \times h_2}{A}\ (\rho_2 g - \rho_1 g) + h_2\rho_2 g - h_1\rho_1 g$

[as A >>> a, so a/A is very very small & can be neglected] -

$\Rightarrow \boxed{P_A = h_2\rho_2 g - h_1\rho_1 g.}$

– Inclined Single Column Manometer

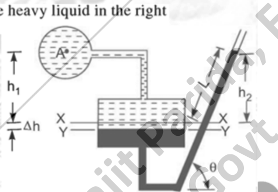
This manometer is more sensitive.

Due to inclination the distance moved by the heavy liquid in the right limb will be more.

Pressure at A, $P_A = h_2\rho_2 g - h_1\rho_1 g$

$\Rightarrow \boxed{P_A = L\sin\theta \times \rho_2 g - h_1\rho_1 g}$

$h_2 = L\sin\theta$



Inclined single column manometer.

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Fluid Pressure and Its Measurements

b) DIFFERENTIAL MANOMETERS

– are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes.

– A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured.

i) U-tube Differential Manometer

Pressure above *X-X* in the left limb $= \rho_1 g(h + x) + p_A$

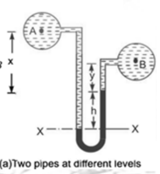
Pressure above *X-X* in the right limb $= \rho_g \times g \times h + \rho_2 \times g \times y + p_B$

Equating the two pressure, we have

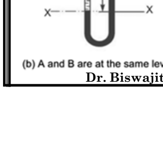
$$\rho_1 g(h + x) + p_A = \rho_g \times g \times h + \rho_2 g y + p_B$$

$$p_A - p_B = \rho_g \times g \times h + \rho_2 g y - \rho_1 g(h + x)$$

$$= h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$$



(a)Two pipes at different levels



(b) A and B are at the same level

Pressure above *X-X* in right limb $= \rho_g \times g \times h + \rho_1 \times g \times x + p_B$

Pressure above *X-X* in left limb $= \rho_1 \times g \times (h + x) + p_A$

Equating the two pressure

$$\rho_g \times g \times h + \rho_1 g x + p_B = \rho_1 \times g \times (h + x) + p_A$$

$$\therefore\ p_A - p_B = \rho_g \times g \times h + \rho_1 g x - \rho_1 g(h + x)$$

$$= g \times h(\rho_g - \rho_1).$$

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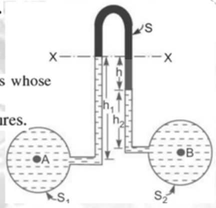
Fluid Pressure and Its Measurements

ii)Inverted U-tube Differential Manometer.



- It consists of an inverted U-tube, containing a light liquid.
- The two ends of the tube are connected to the points whose difference of pressure is to be measured.
- It is used for measuring difference of low pressures.

The pressure in the left limb below X-X
$= p_A - \rho_1 \times g \times h_1$.

Pressure in the right limb below X-X
$= p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$

Equating the two pressure

$$p_A - \rho_1 \times g \times h_1 = p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

$$p_A - p_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h.$$

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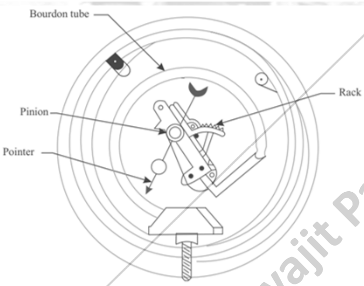
Fluid Pressure and Its Measurements

Bourdon tube pressure gauge:

- is used for measuring high as well as low pressures.
- In this case, the pressure element consists of a metal tube of approximately elliptical cross-section.
- This tube is bent in the form of a segment of a circle and responds to pressure changes.



Bourdon tube pressure gauge.

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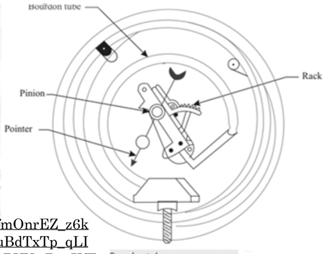
Fluid Pressure and Its Measurements

- When one end of the tube which is attached to the gauge case, is connected to the source of pressure, the internal pressure causes the tube to expand, whereby circumferential stress *i.e.*, hoop tension is set up.
- The free end of the tube moves and is in turn connected by suitable levers to a rack, which engages with a small pinion mounted on the same spindle as the pointer.
- Thus the pressure applied to the tube causes the rack and pinion to move.
- The pressure is indicated by the pointer over a dial which can be graduated in a suitable scale.
- The Bourdon tubes are generally made of *bronze or nickel steel.* The *former* is generally used for *low pressures* and the *latter for high pressures.*



Bourdon tube pressure gauge.

https://www.youtube.com/watch?v=fmOnrEZ_z6k
https://www.youtube.com/watch?v=uBdTxTp_qLI
https://www.youtube.com/watch?v=bRYVwDsxKfE

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REVISING

CHAPTER-02:

FLUID PRESSURE AND ITS MEASUREMENTS

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CHAPTER-03:
HYDROSTATICS

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Hydrostatics

Hydrostatics :-

→ means the study of pressure exerted by the fluid at rest.

→ the direction of such a pressure is always right angle to the surface on which it acts.

Total Pressure :-

→ is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with surface.

→ This force always acts normal to the surface.

Center of Pressure : is the point of application of the total pressure on the surface. There are four cases of submerged surface. (Vertical, Horizontal, Inclined & Curved)

Center of Pressure

a) VERTICAL PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane vertical surface of arbitrary shape immersed in a liquid

Let A = Total area of the surface

$\bar{h}$ = Distance of C.G. of the area from free surface of liquid

G = Centre of gravity of plane surface

P = Centre of pressure

h^* = Distance of centre of pressure from free surface of liquid

Area of the strip, $dA = b \times dh$

Pressure intensity on the strip, $p = \rho g h$

Total pressure force on strip, $dF = p \times \text{Area} = \rho g h \times b \times dh$

$\therefore$ Total pressure force on the whole surface,

$$F = \int dF = \int \rho g h \times b \times dh = \rho g \int b \times h \times dh = \rho g \int h \times dA$$

But $= \int h \times dA$

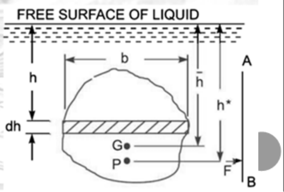
= Moment of surface area about the free surface of liquid

= Area of surface × Distance of C.G. from free surface

$= A \times \bar{h}$

$$\boxed{F = \rho g A \bar{h}}$$

Total Pressure (F)



Center of Pressure

Centre of Pressure (h*) Centre of pressure is calculated by using the "Principle of Moments", which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

moment of the force F about free surface of the liquid $= F \times h^*$

Moment of force dF, acting on a strip about free surface of liquid

$$= dF \times h \quad \{\because \quad dF = \rho g h \times b \times dh\} \quad = \rho g h \times b \times dh \times h$$

Sum of moments of all such forces about free surface of liquid

$$= \int \rho g h \times b \times dh \times h = \rho g \int b \times h \times h dh = \rho g \int b h^2 dh = \rho g \int h^2 dA \quad (\because \quad bdh = dA)$$

$$= \rho g I_0$$

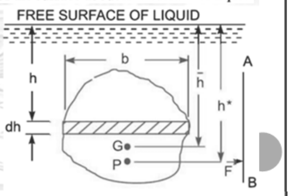
$\because \quad I_0 = \int h^2 dA = \int b h^2 dh$ = Moment of Inertia of the surface about free surface of liquid

$\therefore$ Sum of moments about free surface $= \rho g I_0$

Equating $F \times h^* = \rho g I_0$

But $F = \rho g A \bar{h} \quad \therefore \rho g A \bar{h} \times h^* = \rho g I_0$

$$\boxed{h^* = \frac{\rho g I_0}{\rho g A \bar{h}} = \frac{I_0}{A \bar{h}}}$$



Center of Pressure

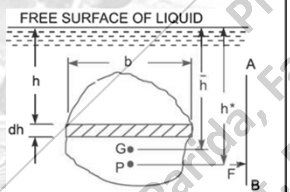
By the theorem of parallel axis, we have

$$I_0 = I_G + A \times \bar{h}^2$$

where I_G = Moment of Inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of liquid.

Substituting I_0 in equation ,

$$\boxed{h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h}}$$

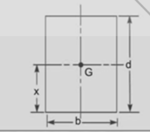
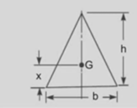


Hence from equation , it is clear that :

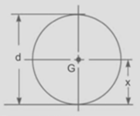
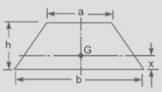
(*i*) Centre of pressure (*i.e.*, h^*) lies below the centre of gravity of the vertical surface.

(*ii*) The distance of centre of pressure from free surface of liquid is independent of the density of the liquid.

The moments of inertia and other geometric properties

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base (I_G)*	*Moment of inertia about base (I_0)*
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$

The moments of inertia and other geometric properties

Plane surface	C.G. from the base	Area	Moment of inertia about an axis passing through C.G. and parallel to base (I_G)	Moment of inertia about base (I_0)
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4. Trapezium 	$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{(a+b)}{2} \times h$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^3$	—

Center of Pressure

b) HORIZONTAL PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane horizontal surface immersed in a static fluid.

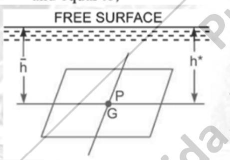
As every point of the surface is at the same depth from the free surface of the liquid, the pressure intensity will be equal on the entire surface and equal to,

$p = \rho g h$, where h is depth of surface.

total force, F, on the surface

$= p \times \text{Area} = \rho g \times h \times A = \rho g A \bar{h}$

$$\boxed{F = \rho g A \bar{h}}$$



where $\bar{h}$ = Depth of C.G. from free surface of liquid = h

also h^* = Depth of centre of pressure from free surface = h.

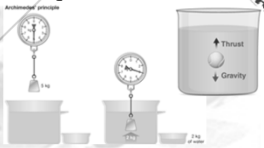
A = Total area of surface

Archimedes Principle

When a body is immersed in a fluid whether wholly or partially, it is buoyed or lifted up by a force, which is equal to the weight of the liquid displaced by the body.

https://www.youtube.com/watch?v=05WkCPORlj4

Buoyancy: - ...be.com/watch?v=XfkJ7wBT-PA



→ whenever a body is immersed wholly or partially in a fluid, it is subjected to an upward force which tends to lift it up.

→ This tendency for an immersed body to be lifted up in the fluid due to an upward force opposite to action of gravity is known as buoyancy this upward force is known as force of buoyancy.

Centre of Buoyancy:

→ is defined as the point through which the forceat of buoyancy is supposed to act.

→ The force of buoyancy is a vertical force & is equal to the wt. of the fluid displaced by the body.

→ Centre of buoyancy will be the CG of the fluid displaced.

Meta-center

- It is defined as the point about which a body starts oscillating when the body is tilted by a small angle.
- may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

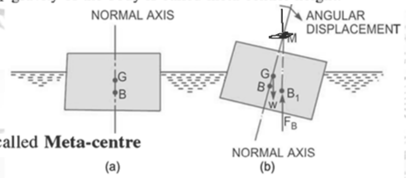
Let the body is given a small angular displacement in the clockwise direction

The centre of buoyancy, which is the centre of gravity of the displaced liquid or centre of gravity of the portion of the body sub-merged in liquid, will now be shifted towards right from the normal axis.

The line of action of the force of buoyancy in this new position

META-CENTRIC HEIGHT

The distance *MG*, *i.e.*, the distance between the meta-centre of a floating body and the centre of gravity of the body is called meta-centric height.



point *M* is called **Meta-centre**

Flotation

Concept of Flotation

Flotation :-

When a body is immersed in any fluid, it experiences two forces

i) wt. of the body W acting vertically downwards.

ii) buoyancy force F_B acting vertically upwards.

For, $W > F_B$ → body will sink in the fluid

$W = F_B$ → body will remain in equilibrium at any level

$W < F_B$ → body will move upwards in the fluid

→ The body moving up will come to rest or stop moving up in the fluid when the fluid displaced by it's submerged part is equal to its weight W, the body in this situation is said to be floating and this phenomenon is known as flotation.

Flotation

Principle of flotation :-

The principle of flotation states that the wt. of the floating body is equal to the wt. of the fluid displaced by the body.

$$F_B = \rho_l V_1 a_1 + \rho_{air} V_2 a_2 = W$$

Since, $\rho_{air} \ll \rho_{liquid}$

$F_B = \rho_{liquid} V_1 a = W$ (Buoyancy force is equal to the wt. of the displaced liquid)

The body can be made to float by.

→ Decreasing the wt. of the body while keeping the volume same, i.e. making body hollow.

→ Increasing the volume of the body while keeping the body same. i.e. attaching live jacket to a person forced the person floating.

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CHAPTER-04: KINEMATICS OF FLOW

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Fluid Kinematics

→ Kinematics is defined as that branch of science which deals with motion of particles without considering the force causing the motion.

→ The velocity at any point in a flow field at any time is studied in this branch of fluid mechanics.

Types of fluid flow :-

i) Steady and unsteady flow :- Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density etc. at a point do not change with time.

So mathematically,

$$\left(\frac{\partial v}{\partial t}\right)_{x_0,y_0,z_0} = 0, \quad \left(\frac{\partial p}{\partial t}\right)_{x_0,y_0,z_0} = 0, \quad \left(\frac{\partial \rho}{\partial t}\right)_{x_0,y_0,z_0} = 0$$

Unsteady flow is that type of flow, in which the velocity, pressure or density at a point changes with respect to time.

$$\left(\frac{\partial v}{\partial t}\right)_{x_0,y_0,z_0} \neq 0, \quad \left(\frac{\partial p}{\partial t}\right)_{x_0,y_0,z_0} \neq 0, \quad \left(\frac{\partial \rho}{\partial t}\right)_{x_0,y_0,z_0} \neq 0$$

Fluid Flow

(i) Uniform & Non-Uniform flow: Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to length of flow or space.

$$\left(\frac{\partial v}{\partial s}\right)_{t=const.} = 0 , \quad \partial s \rightarrow \text{length of flow in the direction } s$$

Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space.

$$\left(\frac{\partial v}{\partial s}\right)_{t=const.} \neq 0$$

(ii) Laminar & Turbulent flow:-

→ Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line & all the stream-lines are straight & parallel.

→ Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called stream-line flow or viscous flow.

Chapter-04: Kinematics of Flow

Fluid Flow

→ Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called stream-line flow or viscous flow.

→ Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called stream-line flow or viscous flow.

→ In turbulent flow the fluid particles move in a zig-zag way due to which eddies formation takes place which are responsible for high energy loss.

$$\text{Reynold Number} = \frac{VD}{\nu}$$

$V \rightarrow$ Mean velocity, $D \rightarrow$ Diameter, $\nu \rightarrow$ Kinematic viscosity.

$R < 2000$ – laminar

$R > 4000$ – turbulent

$2000 < R < 4000$ – may be laminar or turbulent

(iv) Compressible & Incompressible flow: - In compressible flow the density of fluid changes from point to point. $\rho \neq$ constant.

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Chapter-04: Kinematics of Flow

Fluid Flow

For incompressible flow of the fluid, the density is constant. Liquids are generally incompressible while gases are compressible. ρ = constant.

v) Rotational and Irrotational Flow:- In rotational flow the fluid particles while flowing along stream-lines, also rotate about their own axis. And if the fluid particles donot rotate about their own axis while flowing then that type of flow is called irrotational flow.

vi) One, Two & Three dimensional flows:

In One-dimensional flow the flow parameter such as velocity is a function of time & one space co-ordinate only; the variation of velocities in other two mutually perpendicular directions is assumed negligible. Mathematically,

$$u = f(x), \quad v = 0 \ \& \ w = 0$$

where, u, v & w are velocity components in x, y & z directions respectively.

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Fluid Flow

In Two-dimensional flow the flow parameter is a function of time & two rectangular space co-ordinates. The variation of velocity in the third direction is negligible. Mathematically,

$$u = f_1(x, y),\ v = f_2(x, y),\ \&\ w = 0$$

In Three-dimensional flow the velocity is a function of time & three space co-ordinates. Mathematically,

$$u = f_1(x, y, z),\ v = f_2(x, y, z)\ \&\ w = f_3(x, y, z)$$

Rate of flow or discharge :-

→ It is defined as the quantity of a fluid flowing per second through a section of pipe or channel.

$$\boxed{Q = A \times V \rightarrow \text{avg. velocity of the fluid across the section.}}$$

→ For an incompressible fluid (liquid) the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second.

$$Q_{liquid} \rightarrow m^3/s \text{ or } litre/s$$

→ For an compressible fluid (gas) the rate of flow is expressed as the wt. of fluid flowing across the section. $Q_{gas} \rightarrow kgf/s$ or N/s

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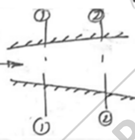
Fluid Kinematics

Equation of Continuity :-



It is based on the principle of conservation of mass and for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is const.

According to law of conservation of mass,

Rate of flow at section 1-1 = Rate of flow at section 2-2

$$\Rightarrow \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

For compressible fluid, then $\rho_1 = \rho_2$

$$\boxed{\Rightarrow A_1 V_1 = A_2 V_2}$$

If no fluid is added removed from the pipe in any length then the mass passing across different sections shall be same.

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Fluid Kinematics

Bernoulli's Equation :-

It states that in a steady ideal flow of an incompressible fluid the total energy at any point of flow is constant.

Mathematically,

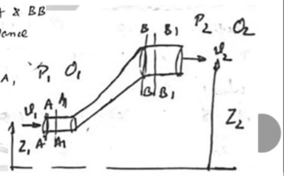
Pressure Energy + Kinetic Energy + Potential or datum energy = const

$$\Rightarrow \frac{p}{\rho g} + \frac{v^2}{2g} + Z = const$$

Derivation :- Consider a perfect incompressible fluid flowing through a non-uniform pipe & it is running full.

Let the liquid between the two sections AA & BB move to AA' & BB' through a very small distance dl_1 & dl_2 (length)

Let W is the wt. of the liquid betwn AA/A_1A_1 & BB/B_1B_1 as the flow is continuous.



$$W = V_1 \rho g = (a_1 \times dl_1) \times w$$

$$\Rightarrow a_1\, dl_1 = \frac{W}{w} = a_2\, dl_2$$

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Fluid Kinematics

$= v_2 \, sg = (O_2 \times dl_2) \times w$

Workdone by pressure to move from AA to A_1A_1 = Force × distance = $P_1 O_1 dl_1$

Work done at BB $= -P_2 O_2 dl_2$

So the total workdone by pressure $= P_1 O_1 dl_1 - P_2 O_2 dl_2$

$= O_1 dl_1 (P_1 - P_2)$ (as volume remains same) $= \frac{W}{w}(P_1 - P_2)$

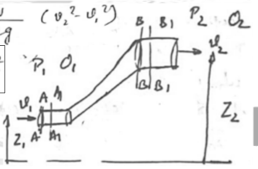
Loss of potential energy $= mg(h_1 - h_2) = W(Z_1 - Z_2)$

Gain in kinetic energy $= \frac{1}{2} m v^2 = \frac{w}{2g}(v_2^2 - v_1^2)$

Loss of PE + Work done by pressure = Gain in KE

$\Rightarrow W(Z_1 - Z_2) + \frac{W}{w}(P_1 - P_2) = \frac{w}{2g}(v_2^2 - v_1^2)$

$$\boxed{\Rightarrow \frac{P_1}{sg} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{sg} + \frac{v_2^2}{2g} + Z_2}$$



Fluid Kinematics

Limitations:-

- Bernoulli's eqⁿ has assumed that the velocity of the fluid particle at any point across the section is uniform
- Loss of energy due to pipe friction during flow of fluid from one section to another are neglected.
- The eqⁿ doesn't take into consideration of loss of energy due to turbulent flow
- It doesn't take into consideration the loss of energy due to change in direction.

Application of Bernoulli's eqⁿ: https://www.youtube.com/watch?v=UJ3-Zm1wbIQ https://www.youtube.com/watch?v=DW4rItB20h4

Bernoulli's eqⁿ is applied in all problems of incompressible fluid flow where energy consideration are involved. It can be applied to following measuring devices.

1) Venturimeter
2) Orifice meter
3) Pitot tube.

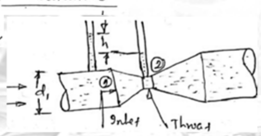
Venturimeter

1. **Venturimeter:-** A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts:
   a) Short converging part
   b) Throat
   c) Diverging part

Expression for rate of flow through venturimeter:

Applying Bernoulli's eqⁿ at sections ① & ②,

$$\frac{P_1}{sg} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{sg} + \frac{v_2^2}{2g} + z_2$$



As the pipe is horizontal, $z_1 = z_2$

$$\Rightarrow \frac{P_1 - P_2}{sg} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

https://www.youtube.com/watch?v=UNBWI6MV_lY

is the difference of pressure heads at sections ① & ② and it is equal to h

Applying continuity eqⁿ at sections ① & ②,

$$a_1 v_1 = a_2 v_2 \Rightarrow v_1 = \frac{a_2 v_2}{a_1}$$

Venturimeter

Substituting the values,

$$h = \frac{v_2^2}{2g} - \frac{(a_2 v_2/a_1)^2}{2g} = \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$\Rightarrow v_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2} \Rightarrow \boxed{v_2 = \frac{a_1}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}}$$

$\therefore$ Discharge, $Q = a_2 v_2 = \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

(discharge under ideal conditions & is called theoritical discharge).

Actual discharge will be less than theoritical discharge,

$$\boxed{Q_{act.} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}}$$

where, $C_d \rightarrow$ co-efficient of venturimeter & its value less then 1

Value of 'h' given by differential U-tube manometer

Case-I :- Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe

S_h & $S_0 \rightarrow$ Sp. Gr. of heavier liquid & liquid flowing through pipe.

Venturimeter

$x \rightarrow$ difference of the heavier liquid column in U-tube

$$P_A - P_B = gx(S_g - S_0) \Rightarrow \frac{P_A - P_B}{S_0 g} = x\left(\frac{S_g}{S_0} - 1\right)$$

$$\Rightarrow \boxed{h = x\left[\frac{S_h}{S_0} - 1\right]}$$

Case-II :- If the differential manometer contains a liquid which is lighter than the liquid flowing through the pipe,

$\boxed{h = x\left[1 - \frac{S_l}{S_0}\right]}$ $S_l, S_0 \rightarrow$ Sp. Gr. of lighter liquid in manometer & fluid flowing through pipe

$x \rightarrow$ Difference in lighter liquid columns in U-tube

Case-III :- is the case for inclined venturimeter having differential U-tube manometer which contains heavier liquid.

$$\boxed{h = \left(\frac{P_1}{Sg} + z_1\right) - \left(\frac{P_2}{Sg} + z_2\right) = x\left(\frac{S_h}{S_0} - 1\right)}$$

Venturimeter

Case-IV :- for inclined venturimeter in which differential manometer contains a liquid which is lighter than the liquid flowing through the pipe,

$$\boxed{h = \left(\frac{P_1}{Sg} + z_1\right) - \left(\frac{P_2}{Sg} + z_2\right) = x\left[1 - \frac{S_l}{S_0}\right]}$$

Limitations :-

→ in actual practice some external forces always acting on the liquid to affect the flow of fluid.

→ If the liquid is flowing in a curved path the energy due to centrifugal force should also be taken into account.

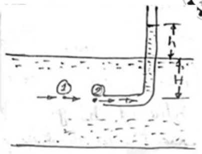
Pitot Tube

2. Pitot-tube:-

→ is a device used for measuring the velocity of flow at any point in a pipe or a channel.

→ it is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the KE into pressure energy.

→ It consists of a glass tube, bent at right angles



Applying Bernoulli's eqn at points ① & ②,

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$z_1 = z_2$ as points are in same line & $v_2 = 0$

https://www.youtube.com/watch?v=3zEdtkuNYLU
https://www.youtube.com/watch?v=wBXqF2Z3L7g
https://www.youtube.com/watch?v=oUd4WxjoHKY

$\frac{P_1}{\rho g}$ = Pr. head at ① = H $\frac{P_2}{\rho g}$ = Pr. head at ② = (h + H)

Substituting the values we get,

$H + \frac{v_1^2}{2g} = (h+H) \Rightarrow v_1 = \sqrt{2gh}$ (Theoretical)

$(v_1)_{act.} = C_v \sqrt{2gH}$ (Actual)

C_v → Co-efficient of pitot-tube

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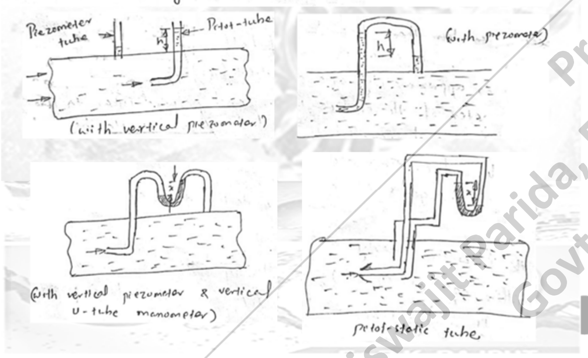
Pitot Tube

For finding the velocity at any point in a pipe by pitot tube, the following arrangements are adopted.



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CHAPTER-05:

ORIFICES, NOTCHES & WEIRS

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Orifices

→ is a small opening of any cross-section (such as circular, triangular, rectangular etc.) on the side of or at the bottom of a tank, through which the fluid is flowing.

→ are used for measuring the rate of flow of fluid.

Classification

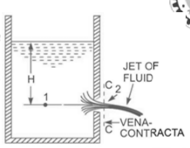
a) Depending upon the size of orifice & head of liquid from the centre of orifice.
→ Small (if the head of liquid above the centre of orifice is more than 5 times the depth of orifice)
→ Large (if head is less than 5 times the depth of orifice

b) Depending upon cross-sectional area.
→ Circular
→ Triangular
→ Rectangular
→ Square

c) Depending upon the shape of upstream edge of the orifices
→ sharp-edged
→ Bell-mouthed

d) Depending upon the nature of discharge
→ Free discharge
→ Drowned or submerged
  → Partially submerged
  → Fully submerged.

Orifices



Flow through Orifice: The area of jet of fluid goes on decreasing and at section C-C, the area is minimum. This section is approximately at a distance of half of diameter of the orifice where the streamlines are straight and parallel to each other and perpendicular to the plane of the orifice. This section is called Vena-contracta beyond which the jet diverges & is attracted in the downward direction by the gravity.

Let the flow is steady & at a constant head H, applying Bernoulli's eq^n at ① & ②, $\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$

$z_1 = z_2$ $\quad$ $v_1 <<< v_2$ as area of tank is very large as compared to the area of the jet of liquid.

$P_1/\rho g = H$

$P_2/\rho g = 0$ (atmospheric pressure) $\therefore H + 0 = 0 + \frac{v_2^2}{2g} \Rightarrow \boxed{v_2 = \sqrt{2gH}}$ (Theoretical velocity)

Actual velocity will be less than the theoretical velocity

Orifices

Hydraulic Co-efficients:-

i) Co-efficient of velocity (Cv):- is defined as the ratio between the actual velocity of a liquid jet at vena-contracta & the theoritical velocity of jet. Mathematically,

$$C_v = \frac{\text{Actual jet velocity at vena-contracta}}{\text{Theoritical velocity}} = \frac{V}{\sqrt{2gH}}$$

Depending on shape, size of the orifice & on the head under which flow takes place value of C_v is between 0.95 to 0.99. Generally 0.98 is for sharp-edged orifices.

ii) Co-efficient of contraction (C_c) :- is defined as the ratio of the area of the jet at vena-contracta to the area of the orifice.

$$C_c = \frac{\text{area of jet at vena-contracta}}{\text{area of orifice}} = \frac{a_c}{a}$$

value of C_c is between 0.61 to 0.69, generally 0.64

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Chapter-05: Orifices, Notches & Weirs

Orifices

iii) Co-efficient of Discharge (Cd): is defined as the ratio of the actual discharge from an orifice to the theoritical discharge from the orifice.

$$C_d = \frac{Q_{act.}}{Q_{th.}} = \frac{\text{Actual velocity} \times \text{Actual area}}{\text{Theoritical velocity} \times \text{Theoritical area}}$$

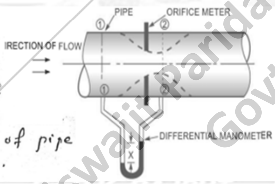
$$= \frac{\text{Actual velocity}}{\text{Theoritical velocity}} \times \frac{\text{Actual area}}{\text{Theoritical area}} \Rightarrow \boxed{C_d = C_v \times C_c}$$

The value of Cd varies from 0.61 to 0.65, generally 0.62

Orifice Meter or Orifice Plate:-

→ is a device used for measuring the rate of flow through a pipe & cheaper as compared to venturimeter

→ The orifice diameter is ~~kept~~ generally 0.5 times the diameter of pipe which may vary from 0.4 to 0.8.



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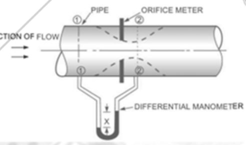
Chapter-05: Orifices, Notches & Weirs

Orifices

Applying Bernoulli's eqn at ① & ②



$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\Rightarrow \underbrace{\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right)}_{= h = \text{differential head.}} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$\Rightarrow h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \Rightarrow v_2^2 = \sqrt{2gh + v_1^2}$$

a_2 → area at vena-contracta, a_0 → area of orifice.

So, Co-efficient of contraction $= C_c = \frac{a_2}{a_0} \Rightarrow a_2 = a_0 \times C_c$

By continuity eqn we have,

$$a_1 v_1 = a_2 v_2 \Rightarrow v_1 = \frac{a_2 v_2}{a_1} = \frac{a_0 C_c}{a_1} v_2$$

$$\Rightarrow v_2 = \sqrt{2gh + \frac{a_0^2 C_c^2 v_2^2}{a_1}} \Rightarrow v_2^2 \left[1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2\right] = 2gh$$

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Orifices

$$\Rightarrow v_2 = \sqrt{2gh} \Big/ \sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}$$

The discharge, $Q = v_2 \times a_2 = v_2 \times a_0 C_c$

$$= a_0 C_c \sqrt{2gh} \Big/ \sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}$$

$$C_d = C_c \sqrt{1-(a_0/a_1)^2} \Big/ \sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2} \qquad C_c = C_d \sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2} \Big/ \sqrt{1-\left(\frac{a_0}{a_1}\right)^2}$$

– Substituting this value,

$$Q = a_0 \times C_d \frac{\sqrt{1-(a_0/a_1)^2 C_c^2}}{\sqrt{1-(a_0/a_1)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$= \frac{C_d a_0 \sqrt{2gh}}{\sqrt{1-(a_0/a_1)^2}} = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

where, C_d ⇒ coefficient of discharge for orifice meter; which is much smaller than for a venturimeter

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Notches & Weirs

Notch:-

→ is a device used for measuring the rate of flow of a liquid through a small channel.

→ is an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

→ Small in size & made of metallic plate

Weir:

https://www.youtube.com/watch?v=hjSaSkto7_c
https://www.youtube.com/watch?v=gxJWAUqGX9w

→ It is a concrete or masonry structure, placed in an open channel over which the flow occurs.

→ in the form of vertical wall, with a sharp edge at the top, running all the way across the open channel.

→ Big in size and made of concrete or masonry structure

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Notches & Weirs

Other terminologies:

a) Nappe or Vein: the sheet of water flowing through a notch or over a weir.

b) Crest or Sill: the bottom edge of a notch or a top of a weir over which the water flows.

Classification of Notches & Weirs

Notches	Weirs
a) According to the shape of the opening:-	
→ Rectangular Notch	→ Rectangular weir
→ Triangular Notch	→ Triangular weir
→ Trapezoidal Notch	→ Trapezoidal weir (Cipolletti weir)
→ Stepped Notch	
b) According to the effect of the sides on the nappe:	
→ Notch with end contraction	→ Weir with end contraction
→ Notch without end contraction or Suppressed Notch	→ Weir without end contraction
	c) According to the shape of the crest
	→ Sharp crested weir → Broad crested weir
	→ Narrow-crested weir → Ogee-shaped weir

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Notches & Weirs

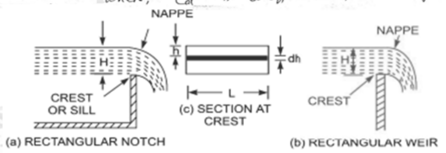
DISCHARGE OVER A RECTANGULAR NOTCH OR WEIR

Consider a rectangular notch or weir having,
$H \rightarrow$ Head of water over the crest
& $L \rightarrow$ Length of the notch or weir.
Let us consider a horizontal strip of water of thickness dh & length L at a depth h from the free surface.

Area of the strip $= L \times dh$ Water velocity $= V_{th.} = \sqrt{2gh}$

Discharge dQ, through strip

$$dQ = C_d \times \text{Area} \times V_{th.}$$
$$= C_d \times L \times dh \times \sqrt{2gh}$$ where, $C_d \rightarrow$ Co-efficient of discharge



(a) RECTANGULAR NOTCH (b) RECTANGULAR WEIR

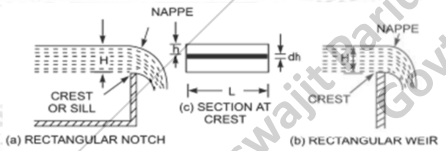
Notches & Weirs

So the total discharge,

$$Q = \int_0^H C_d \cdot L \cdot \sqrt{2gh} \cdot dh = C_d \times L \times \sqrt{2g} \int_0^H h^{1/2} dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{1/2+1}}{1/2+1} \right]_0^H = C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H$$

$$\Rightarrow \boxed{Q = \tfrac{2}{3} C_d \times L \times \sqrt{2g}\, [H]^{3/2}}$$

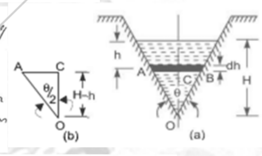


(a) RECTANGULAR NOTCH (b) RECTANGULAR WEIR

Notches & Weirs

DISCHARGE OVER A TRIANGULAR NOTCH OR WEIR

Consider a triangular notch or weir having
$H \rightarrow$ head of water above the v-notch
$\theta \rightarrow$ Angle of notch
Let us consider a horizontal strip of water of thickness 'dh' at a depth of h from the free surface of water.



We have, $\tan \theta/2 = \frac{AC}{OC} = \frac{AC}{(H-h)} \Rightarrow AC = (H-h) \tan \theta/2$

width of strip $= AB = 2AC = 2(H-h) \tan \theta/2$

$\therefore$ Area $= 2(H-h) \tan \theta/2 \times dh$

velocity through strip $= V_{th.} = \sqrt{2gh}$

Discharge through the strip,

$$dQ = C_d \times \text{Area} \times V_{th.}$$
$$= C_d \times 2(H-h) \tan \theta/2 \times dh \times \sqrt{2gh}$$

Notches & Weirs

∴ Total discharge,

$$Q = \int_0^H 2\,C_d\,(H-h)\tan\theta/2 \times \sqrt{2gh}\,dh$$

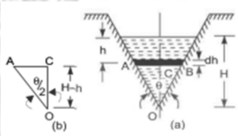
$$= 2\,C_d \times \tan\theta/2 \times \sqrt{2g}\int_0^H (H-h)\,h^{1/2}\,dh$$

$$= 2 \times C_d \times \tan\theta/2 \times \sqrt{2g}\int_0^H \left[H h^{1/2} - h^{3/2}\right] dh$$

$$= 2 \times C_d \times \tan\theta/2 \times \sqrt{2g}\left[2/3\,H \cdot H^{3/2} - 2/5\,H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\theta/2 \times \sqrt{2g}\left[\frac{4}{15}H^{5/2}\right]$$

$$\Rightarrow \boxed{Q = 8/15\,C_d \times \tan\theta/2 \times \sqrt{2g} \times H^{5/2}}$$



For a right angled V-notch, if $C_d = 0.6$, $\theta = 90^\circ \Rightarrow \tan\theta/2 = 1$

$$\Rightarrow Q = \text{Discharge} = 8/15 \times 0.6 \times 1 \times \sqrt{2 \times 9.81} \times H^{5/2}$$

$$\Rightarrow \boxed{Q = 1.417\,H^{5/2}}$$

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https://www.youtube.com/watch?v=6V3Ctem6hdM

CHAPTER-06: FLOW THROUGH PIPE

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Pipe

Pipe :-

→ It is a closed conduit, generally of circular cross-section used to carry water or any other fluid.

→ When the pipe is running full, the flow is under pressure but if not the flow is not under pressure culverts, sewer pipes.

Loss of fluid friction :-

→ The frictional resistance of a pipe depends upon the roughness of the inside surface of the pipe. This friction is known as fluid friction & the resistance is known as frictional resistance.

→ According to Froude, the frictional resistance varies with the square of the velocity and nature of the surface.

Classification

Energy Losses

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost.

1. Major Energy Losses

This is due to friction and it is calculated by the following formulae :
(*a*) Darcy-Weisbach Formula
(*b*) Chezy's Formula

2. Minor Energy Losses

This is due to
(*a*) Sudden expansion of pipe
(*b*) Sudden contraction of pipe
(*c*) Bend in pipe
(*d*) Pipe fittings etc.
(*e*) An obstruction in pipe.

Loss of Energy (Minor Losses)

Darcy-Weisbach Formula :-

The loss of head or energy in pipes due to friction is calculated from Darcy-Weisbach equation is given by,

$$h_f = \frac{4fLV^2}{d \times 2g}$$, f = co-eff. of friction which is a function of Reynold's number.

L = length of pipe,
V = mean velocity of flow,
d = diameter of pipe.

$f = \frac{16}{Re}$ for $Re < 2000$ (viscous flow)
$= \frac{0.079}{Re^{1/4}}$ for Re from 4000 to 10^6

Chezy's Formula :-

The loss of head due to friction in pipes are derived from,

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2$$

P → wetted perimeter of pipe, A → cross-sectional area of pipe, L → length of pipe, V → mean velocity of flow.

Loss of Energy (Minor Losses)

Hydraulic radius / hydraulic mean depth,

$$m = \frac{\text{Area of flow}}{\text{Perimeter (wetted)}} = \frac{A}{P} = \frac{\frac{\pi}{4}d^2}{\pi d} = d/4$$

$$\Rightarrow h_f = \frac{f'}{\rho g} \times L \times v^2 \times \frac{1}{m} \Rightarrow v^2 = h_f \times \frac{\rho g}{f'} \times m \times \frac{1}{L}$$

$$\Rightarrow \boxed{v = \sqrt{\frac{\rho g}{f'}}\sqrt{m\frac{h_f}{L}}}$$

Let $\sqrt{\frac{\rho g}{f'}} = C$, where C is a constant known as Chezy's constant &

$\frac{h_f}{L} = i$, where i is the loss of head per unit length of pipe,

Substituting the value, $\boxed{v = C\sqrt{mi}} \rightarrow$ Chezy's formula

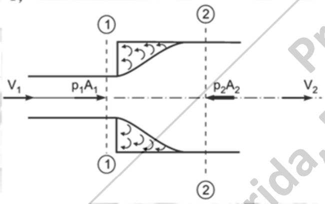
The loss of head due to friction in pipe from Chezy's formula can be obtained if the velocity of flow through pipe & also the value of C is known. The value of m for pipe is always $d/4$.

Loss of Energy (Minor Losses)

Loss of Head Due to Sudden Enlargement

Let
p_1 = pressure intensity at section 1-1,
V_1 = velocity of flow at section 1-1,
A_1 = area of pipe at section 1-1,

p_2, V_2 and A_2 = corresponding values at section 2-2.



Let
p' = pressure intensity of the liquid eddies on the area $(A_2 - A_1)$
h_e = loss of head due to sudden enlargement

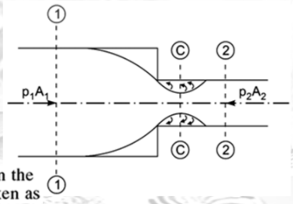
$$\boxed{h_e = \frac{(V_1 - V_2)^2}{2g}}.$$

Loss of Energy (Minor Losses)

Loss of Head due to Sudden Contraction.

Let A_c = Area of flow at section C-C
V_c = Velocity of flow at section C-C
A_2 = Area of flow at section 2-2
V_2 = Velocity of flow at section 2-2
h_c = Loss of head due to sudden contraction.

$$\boxed{h_c = \frac{kV_2^2}{2g} = 0.375\,\frac{V_2^2}{2g}}$$



If the value of C_c is not given then the head loss due to contraction is taken as

$$\boxed{= 0.5\,\frac{V_2^2}{2g} \text{ or } h_c = 0.5\,\frac{V_2^2}{2g}}.$$

Loss of Energy (Minor Losses)

Loss of Head at the Entrance of a Pipe.

- This is the loss of energy which occurs when a liquid enters a pipe which is connected to a large tank or reservoir.
- This loss is similar to the loss of head due to sudden contraction.
- This loss depends on the form of entrance.

$$\boxed{h_i = 0.5\,\frac{V^2}{2g}}$$ where V = velocity of liquid in pipe.

Loss of Head at the Exit of Pipe.

This is the loss of head (or energy) due to the velocity of liquid at outlet of the pipe which is dissipated either in the form of a free jet (if outlet of the pipe is free) or it is lost in the tank or reservoir (if the outlet of the pipe is connected to the tank or reservoir).

$$\boxed{h_o = \frac{V^2}{2g}}$$ where V = velocity at outlet of pipe.

Loss of Energy (Minor Losses)

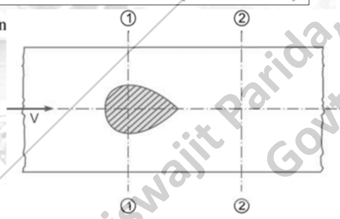
Loss of Head Due to an Obstruction in a Pipe.

Whenever there is an obstruction in a pipe, the loss of energy takes place due to reduction of the area of the cross-section of the pipe at the place where obstruction is present.

Let a = Maximum area of obstruction
A = Area of pipe
V = Velocity of liquid in pipe

$$\boxed{\text{Head loss due to obstruction} = \frac{V^2}{2g}\left(\frac{A}{C_c\,(A-a)} - 1\right)^2}$$

C_c = co-efficient of contraction



Loss of Energy (Minor Losses)

Loss of Head due to Bend in Pipe.

When there is any bend in a pipe, the velocity of flow changes, due to which the separation of the flow from the boundary and also formation of eddies takes place.

$$\boxed{h_b = \frac{kV^2}{2g}}$$

where h_b = loss of head due to bend, V = velocity of flow, k = co-efficient of bend

The value of k depends on

(*i*) Angle of bend, (*ii*) Radius of curvature of bend, (*iii*) Diameter of pipe.

Loss of Head in Various Pipe Fittings.

The loss of head in the various pipe fittings such as valves, couplings etc. is expressed as

$$\boxed{= \frac{kV^2}{2g}}$$

where V = velocity of flow, k = co-efficient of pipe fitting.

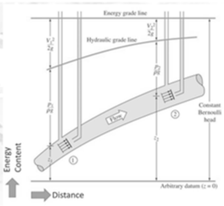
Loss of Energy (Minor Losses)

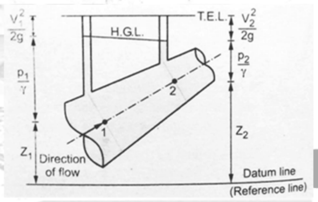
Hydraulic Gradient Lines. (HGL)

It is defined as the line which gives the sum of pressure head ($P/\rho g$) and datum head (z) of a flowing fluid in a pipe with respect to some reference line.

or

It is the line which is obtained by joining the tops of all vertical ordinates, showing the pressure head of a flowing fluid in a pipe from the centres of the pipe.





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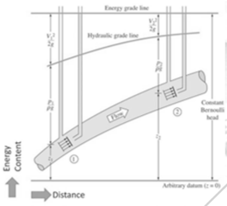
Loss of Energy (Minor Losses)

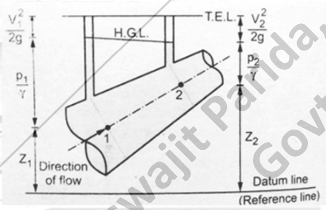
Total Energy Line :- (TEL)

It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line.

or

It is also defined as the line which is obtained by joining the top of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe.





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Chapter-07: Impact of Jets

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Impact of Jets

→ Impact of jet means the force exerted by the liquid jet coming from the outlet of a nozzle, which is fitted to a pipe through which the liquid is flowing under pressure) on a plate which may be stationary or moving.

https://www.youtube.com/watch?v=fHepVMqITVo
https://www.youtube.com/watch?v=0xXoev-gaas

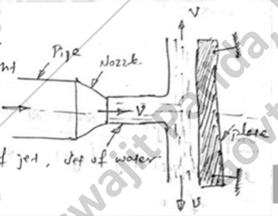
→ This force is obtained from Newton's second law of motion or from impulse-momentum equation.

Impact of jet on fixed vertical flat plate:-

v → jet velocity, $a = \frac{\pi}{4} d^2$

a, d → area & diameter of the jet

As the plate is fixed and is at right angles to the jet, after striking it will deflected through 90° for which the component of the velocity of jet in the direction of jet, after striking will be zero.



The force exerted by the jet on the plate in the direction of jet,

F_x = Rate of change of momentum in the direction of force

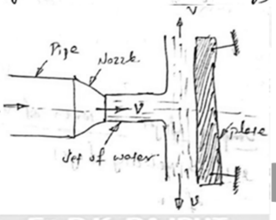
$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$= \frac{(\text{Mass} \times \text{Initial velocity}) - (\text{Mass} \times \text{Final velocity})}{\text{Time}}$$

$= (\text{mass/sec}) \times (\text{velocity of jet before striking} - \text{velocity of jet after striking})$

$\left(\rho \times \text{Area} \times \frac{\text{length}}{\text{sec}}\right) \, (v - 0)$

$\Rightarrow F_x = \rho a v (v) \Rightarrow \boxed{F_x = \rho a v^2}$



Impact of jet on : vertical flat moving plate :-

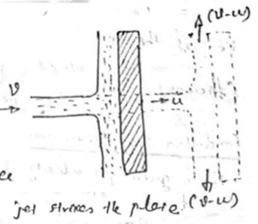
$v \rightarrow$ absolute velocity of the jet.

$u \rightarrow$ velocity of flat plate.

Relative velocity of jet with respect to plate,

= absolute velocity of jet − velocity of plate

$= (v - u)$



Mass of water striking the plate per sec

$= \rho \times$ Area of jet $\times$ velocity with which jet strikes the plate

$= \rho a (v-u)$

Force exerted by the jet on the moving plate in the direction of the jet,

F_x = Mass of water striking per sec. $\times$ [Initial velocity with which water strikes − Final velocity]

$= \rho a (v-u) [(v-u) - 0] \rightarrow \boxed{F_x = \rho a (v-u)^2}$

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For stationary plates the workdone is zero, but for moving plate the work will be done by the jet on the plate,

$= \text{Force} \times \dfrac{\text{Distance in the direction of force}}{\text{Time}}$

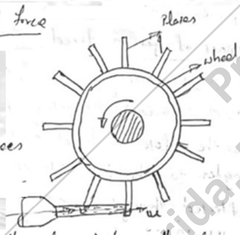
$= F_x \times u = \rho a (v-u)^2 \times u$

Impact of jet on series of vanes :-

$v, u \rightarrow$ velocity of jet & vane

Mass of water per sec. striking the series of plates $= \rho A v$

The jet strikes with a velocity $= (v-u)$



After striking the jet moves tangential to the plate & hence the velocity component in the direction of motion of plate is equal to zero.

So the force exerted by the jet in the direction of motion of the plate,

F_x = Mass per sec. [Initial velocity − Final velocity]

$= \rho a v [(v-u) - 0] = \rho a v (v-u)$

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Workdone by the jet on the series of plates per sec.

= Force $\times$ Distance per sec. in the direction of force

$= F_x \times u = \rho a v (v-u) \times u$

KE of the jet per sec. $= \frac{1}{2} m v^2$

$= \frac{1}{2} (\rho a v) \times v^2 = \frac{1}{2} \rho a v^3$

Efficiency $= \eta = \dfrac{\text{Workdone per sec.}}{\text{KE per sec.}}$

$= \dfrac{\rho a v (v-u) \times u}{\frac{1}{2} \rho a v^3} = \dfrac{2u(v-u)}{v^2}$

Condition for Maximum Efficiency →

For a given jet velocity v, the efficiency will be maximum when,

$\dfrac{d\eta}{du} = 0 \Rightarrow \dfrac{d}{du}\left(\dfrac{2u(v-u)}{v^2}\right) = 0 \Rightarrow \dfrac{2v - 2 \times 2u}{v^2} = 0 \Rightarrow \boxed{u = \dfrac{v}{2}}$

$\therefore \eta_{max} = \dfrac{2u[2u-u]}{(2u)^2} = \frac{1}{2} = 0.5$ or 50%

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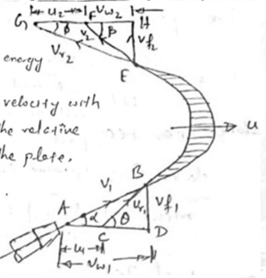
Impact of jet on a curved plate when jet strikes tangentially at one of the tips :-



As the jet strikes tangentially, the loss of energy due to impact of jet will be zero.

In this case plate is moving, so the velocity with which jet of water strikes is equal to the relative velocity of the jet with respect to the plate.

$V_1, u_1 \rightarrow$ velocity of jet & plate at inlet.

$V_{r_1} \rightarrow$ relative " " " "

$\alpha \rightarrow$ guide blade angle

$\theta \rightarrow$ vane or plate angle at inlet.

V_{w_1} & $V_{f_1} \rightarrow$ velocity of whirl and flow at inlet.

$\beta \rightarrow$ angle made by V_2 with the direction of motion of the vane at outlet

$\phi \rightarrow$ vane angle at outlet.

Δ ABD & EGH $\rightarrow$ velocity triangles at inlet & outlet.

If the vane is smooth and is having velocity in the direction of motion at inlet and outlet equal then we have,

$u_1 = u_2 = u$ = velocity of vane in the direction of motion

& $V_{r_1} = V_{r_2}$

Mass of water striking the vane per sec. $= \rho a V_{r_1}$

$\therefore$ Force exerted by the jet in the direction of motion,

$F_x = \frac{\text{Mass}}{\text{sec.}}$ [Initial velocity with which jet strikes in the direction of motion − Final velocity of jet in the direction of motion]

Initial velocity with which jet strikes the vane $= V_{r_1}$

Component of this in direction of motion $= V_{r_1}\cos\theta = (V_{w_1} - u_1)$

Component of the relative velocity at outlet in the direction of motion.

$= -V_{r_2}\cos\phi = -(u_2 + V_{w_2})$

$\Rightarrow F_x = \rho a V_{r_1}[(V_{w_1} - u_1) - \{-(u_2 + V_{w_2})\}] = \rho a V_{r_1}[V_{w_1} + V_{w_2}]$ true for $\beta < 90^\circ$,

if $> 90^\circ$, $F_x = \rho a V_{r_1}[V_{w_1} - V_{w_2}]$

if $= 90^\circ$, $V_{w_2} = 0 \Rightarrow F_x = \rho a V_{r_1}[V_{w_1}]$

So in general, $F_x = \rho a V_{r_1}[V_{w_1} \pm V_{w_2}]$

$\therefore$ Workdone per sec. on the vane by the jet

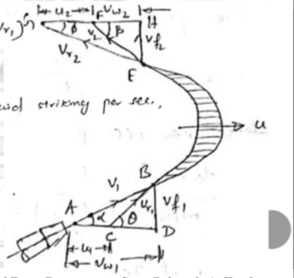
= Force × Distance per second in the direction of force.

$= F_x \times u = \rho a V_{r_1}[V_{w_1} \pm V_{w_2}] \times u$

Work done per sec. per unit wt. of fluid striking per sec.

$= \frac{\rho a V_{r_1}[V_{w_1} \pm V_{w_2}] \times u}{\text{wt. of fluid striking } (g \times \rho a V_{r_1})}$

$= \frac{1}{g}[V_{w_1} \pm V_{w_2}] \times u$



Workdone per sec. per unit mass of fluid striking per sec.

$= \frac{\rho a V_{r_1}[V_{w_1} \pm V_{w_2}] \times u}{\text{mass of fluid striking } (\rho a V_{r_1}) \text{ per sec.}}$

$= (V_{w_1} \pm V_{w_2}) \times u.$

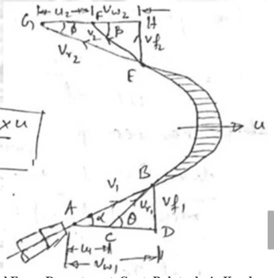
Efficiency of Jet :-

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Work done per sec. on the vane}}{\text{Initial KE per sec. on the jet}}$$

$$= \frac{\rho a V_{r_1} (V_{w_1} \pm V_{w_2}) \times u}{\frac{1}{2} m_1 V_1^2}$$

$$= \frac{\rho a V_{r_1} (V_{w_1} \pm V_{w_2}) \times u}{\frac{1}{2} (\rho a V_1) \times V_1^2}$$

$$\Rightarrow \boxed{\eta = \frac{2 V_{r_1} (V_{w_1} \pm V_{w_2}) \times u}{V_1^2}}$$



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