GOVERNMENT POLYTECHNIC

KENDRAPARA

DEPARTMENT OF CIVIL

ENGINEERING

LECTURE NOTES

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WORKING STRESS METHOD

Objective of design and detailing:

Every structure must be **designed** to satisfy three basic requirements;

- **Stability** to prevent overturning, sliding or buckling of the structure, or parts of it, under the action of load.
- **Strengths** to resist safely the stresses induced by the loads in the various structural members.
- **Serviceability** to ensure satisfactory performance under service load condition. Serviceability includes two parameters i.e deflection and cracking. The deflection should be limited to ensure the better appearance of the structure and to prevent cracking. The cracking of the reinforced concrete should not be excessive to ensure better appearance and also to prevent the access of water from cracks which may corrode the reinforceme

There are two other considerations that a sensible designer ought to bear in mind, viz. **economy** and aesthetics.

Different methods of design:

(a) Working stress method:

- The earliest codified design philosophy is that of **working stress method** of design (WSM).
- Close to a hundred years old, this traditional method of design, based on linear elastic theory is still surviving in a number of countries.
- In WSM it is assumed that structural material e.g. concrete and steel behave in linearly elastic manner and adequate safety can be ensured by restricting the stresses in the material induced by working loads (service loads) on the structure.
- As the specified permissible (allowable) stresses are kept well below the material strength, the assumption of linear elastic behavior considered justifiable.
- The ratio of the strength of the material to the permissible stress is often referred to as the factor of safety.

(b) Ultimate load method:

- In this method the inelastic behavior of concrete is taken into account and therefore reserve strength of concrete can be used leading to the economical design.
- In design, the loads on the structure are increased by suitable load factor and the structure is loaded with these increased loads called ultimate load.
- For material (steel and concrete) the ultimate behavior is taken into account.
- The ultimate load method leads the designer to economy but at the same time, to very slender section, larger deflection and larger crack width in concrete.

(c) Limit state method:

- In this method the design based on limit state concept, the structure shall be designed to withstand safely all loads liable to act on it through out its life.
- It shall also satisfy the serviceability requirement such as prevention of excessive deflection and cracking.
- WSM gives satisfactory performance of the structure at working loads and it is unrealistic at ultimate state of collapse.
- ULM provides realistic assessment of safety but it does not guarantee the satisfactory serviceability requirement at service load.
- The acceptable limit for the safety and serviceability requirement before failure occurs is called "limit state"
- The aim of the design is to achiev acceptable probabilities that the structure will not become unfit for the use for which it is intended, that means it will not reach a limit state. For ensuring the above objective, various partial factor of safety are employed in the limit state design.
- The design values are obtained from the characteristic values through the use of partial factor of safety. Design loads are obtained by multiplying a partial factor of safety for loads with characteristic load and in a similar manner the design strength of material are obtained by dividing the characteristic strength with respective partial factor of safety for materials.

Reinforced cement concrete:

- It is a combination of concrete and steel to build a structure instead of using only concrete.
- Concrete is good in resisting compression but is weak in resisting tension.
- On the otherhand steel has high tensile strength and the bond between concrete and steel is good.
- To overcome the drawback of concrete, steel bars are used alongwith concrete.

Grades of concrete:

- The properties of concrete vary so much with composition and method of mixing therefore different types of concrete can be obtained.
- Concrete mixes have been classified into various grades by the Indian Standard Institution.

Type of concrete	Grade designation	Characteristic compressive strength at 28 days in N/mm^2
Normal strength concrete	M5	5

	M10	10
	M15	15
Standard concrete	M20	20
	M25	25
	M30	30
High strength concrete	M35 onwards	35

- In this designation M refers to mix and the number represents the characteristic compressive strength of cube at 28 days expressed in N/mm^2
- Characteristic strength is defined as the strength of the material below which not more than 5% of the test results are expected to fall.

Advantages of reinforced cement concrete:

- Reinforced Cement Concrete has good compressive strength (because of concrete).
- RCC also has high tensile strength (because of steel).
- It has good resistance to damage by fire and weathering (because of concrete).
- RCC protects steel bars from buckling and twisting at the high temperature
- RCC prevents steel from rusting
- Reinforced Concrete is durable
- The monolithic character of reinforced concrete gives it more rigidity.
- Maintenance cost of RCC is practically nil.

Concept of under, balanced and over reinforced beam section:

Under reinforced section:

- In some cases a larger section provided than required for a balanced section. In otherwords, a smaller area of steel is provided than required for a balanced section.
- In this case at some value of loads, the stress in steel will reach at its permissible or design value and fails while concrete stress is less than its permissible value.
- The failure in this case is a tension or ductile failure.

Balanced reinforced section:

- In this type of design, the section is so proportioned that the steel and concrete both reach their maximum values of stress at the same time. Thus at some value of load both the material will fail at the same time.

Over reinforced:

- In this case steel area provided is more than the area frequired for a balanced section.
- At some value of load the stress in concrete will reach at its permissible value and fails while stress

in steel is less than its permissible value.

- The failure in this case is called compression failure and therefore it will be a brittle failure.

Working stress method:

Assumptions :

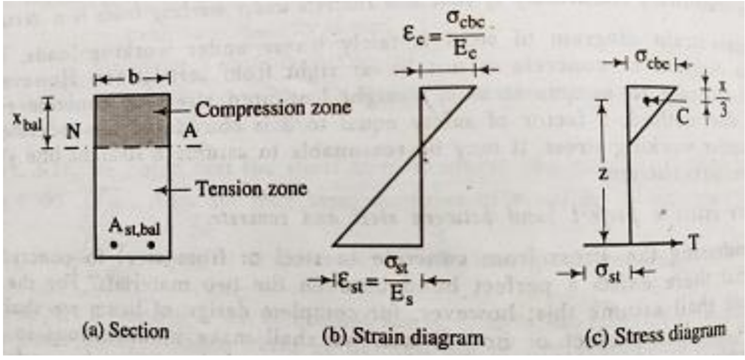
The analysis and design of a RCC member are based on the following assumptions

- Concrete is assumed to be homogeneous
- At any cross section, plane sections before bending remain plane after bending.
- The stress-strain relationship for concrete is a straight line, under working loads
- The stress-strain relationship for steel is a straight line, under working loads
- Concrete area on tension side is assumed to be ineffective
- All tensile stresses are taken up by reinforcements and none by concrete except when specially permitted.
- The steel area is assumed to be concentrated at the centroid of the steel.
- The modular ratio has the value $280/3\sigma_{cbc}$ where σ_{cbc} is permissible stress in compression due to bending in concrete in N/mm^2 as specified in code(IS:456-2000).

Permissible stresses:

- In working stress method, the stresses in materials are not exceeded beyond their permissible values. The permissible stresses are found by using suitable factor of safety to the material strength.
- For concrete in compression in bending a factor of safety equal to 3 is considered on characteristic strength of concrete.
- For steel a factor of safety equal to 1.8 is considered on a yield strength of steel in tension due to bending.

Derivation of formula for balanced design: consider a singly reinforced beam with stress and strain diagram as shown in figure.



Ast,bal = reinforcement area provided for balanced section

Ơcbc = permissible stress in concrete in bending compression

Ơst = permissible stress in steel in tension

Ec = modulus of elasticity of concrete

Est = modulus of elasticity in steel

€c = strain in concrete in extreme compression fibre = ơcbc/Ec

€st = strain in steel = ơst/Es = ơst/mEc

b = width of beam

d = effective depth which is defined as the distance from extreme compression fibre to the centroid of tensile reinforcement

x = depth of N.A which is defined as the distance of neutral axis from extreme compression fibre.

z = lever arm which is defined as the distance between centroid of compressive force to the centroid of tensile force

To find Neutral axis :

From the strain diagram

$$\frac{xbal}{d-xbal} = \frac{\sigma cbc/Ec}{\sigma st/Es} = \frac{m\sigma cbc}{\sigma st}$$

$$X_{bal} = \frac{m\sigma cbc}{m\sigma cbc+\sigma st}d = \frac{1}{1+\frac{\sigma st}{m\sigma cbc}} d$$

X_{bal}= kd

Where $k = \frac{1}{1+\frac{\sigma st}{m\sigma cbc}}$

and is known as neutral axis constant

To find lever arm:

From stress diagram

$$Z = d - \frac{xbal}{3}$$

$$= d - \frac{kd}{3} = (1-\frac{k}{3})d$$

$$= jd$$

Where the constant $j = (1 - \frac{k}{3})$

And is known as lever arm constant

To find total forces:

C = total compression

T = total tension

$C = ½\sigma cbc\ bx_{bal}$

$= \frac{bxbal\sigma cbc}{2}$

$T = \sigma_{st}A_{st,bal}$

To find moment of resistance of section

Capacity of a section to resist the moment is known as its moment of resistance.

M.R = total compressive force x lever arm

Or,

M.R = total tensile force x lever arm

Considering compressive force

M.R = total compression x lever arm

$= (½\ \sigma_{cbc}bx_{bal})jd$

$= (½\sigma_{cbc}bkd)jd$

$= (½\sigma_{cbc}kj)bd^2$

$M = Q_{bal}bd^2$

Where $Q_{bal} = ½\sigma_{cbc}kj$ and is known as moment of resistance factor

Considering the tensile forces

M.R = total tension x lever arm

$= A_{stbal}\sigma_{st}jd$

To find steel area

For a balanced section

$M_{bal} = A_{stbal}\sigma_{st}jd$

$A_{stbal} = \frac{M_{bal}}{\sigma_{st}\ jd}$

Percentage steel = $p_t = 100A_{st}/bd$

For a balanced section $p_{tbal} = 100 \times \frac{Astbal}{bd} = 100 \times \frac{Mbal}{\sigma_{st}jd} \times \frac{1}{bd}$

$$P_{tbal} = \frac{100 \times ½\sigma cbc \times k \times j \times bd \times d}{\sigma st \times jd \times bd} = \frac{50\sigma cbc \times k}{\sigma st}$$

To design balanced section

For a given design moment M, consider $M = M_{bal}$

If width of beam is assumed

$d = \sqrt{\frac{M_{bal}}{bQ_{bal}}}$

steel area = A_{st} = A_{stbal} = $\frac{M}{\sigma_{st} jd}$

Question 1

Calculate the design constants for the following material considering the balanced design for singly reinforced section. The materials are M20 grade concrete and mild steel reinforcement.

Solution:

For M20 grade concrete, σ_{cbc} = 7N/mm^2

For Fe250 grade steel, σ_{st} = 140 N/mm^2

Modular ratio = m = $\frac{280}{3\sigma_{cbc}}$ = $\frac{280}{3\times7}$ = 13.33

Neutral axis constant = k = $\frac{1}{1+\frac{\sigma st}{m\sigma cbc}}$ = $\frac{1}{1+\frac{140}{13.33\times7}}$

= 0.4

Lever arm constant = j = $(1 - \frac{k}{3})$ = $(1 - \frac{0.4}{3})$

= 0.87

M.R constant = Q_{bal} = ½σ_{cbc}kj = ½×7×0.4×0.87

= 1.21

P_{tbal} = $\frac{50\sigma cbc\times k}{\sigma st}$ = $\frac{50\times7\times0.4}{140}$ = 1.0

Question 2 :

For a rectangular beam of size 250 mm wide x 520 mm effective depth, find out the balanced depth of neutral axis, balanced lever arm, balanced moment of resistance and balanced steel area. The materials are M20 grade concrete and HYSD reinforcement of grade Fe 415.

Solution:

b = 250 mm

d = 520 mm

σcbc = 7 N/mm^2

σst = 230 N/mm^2

k = 0.29

j = 0.90

Q_{bal} = 0.91

Depth of neutral axis = kd = 0.29 × 520 = 150.8 mm

Lever arm = jd = 0.9 × 520 = 468 mm

M.R of balanced section = M = $Q_{bal}bd^2$ = 0.91×250×520×520×10^{-6} = 61.52 KNm

A_{stbal} = $\frac{p_{tbal}\times bd}{100}$ = $\frac{0.44\times250\times520}{100}$ = 572 mm^2

Question 3

A simply supported rectangular beam of 4m span carries a uniformly distributed load of 26 KN/m . The width of the beam is 230 mm. find the depth and steel area for balanced design. Use M20 grade concrete and mild steel reinforcement.

Solution :

Maximum moment M = $26\times4^2/8$

= 52 Kn.m

For balanced section Q_{bal} = 1.21

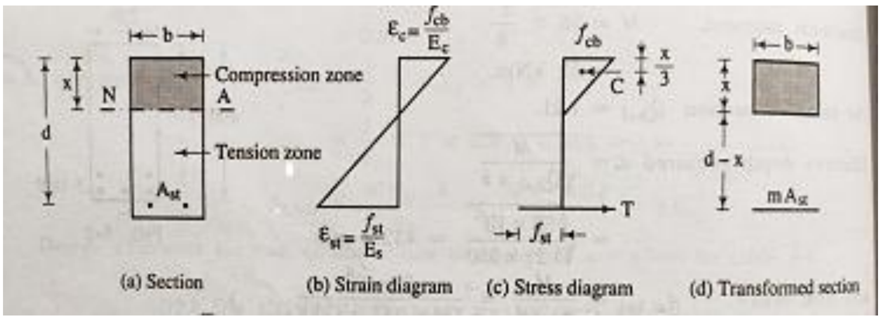
Effective depth required = d = $\sqrt{\frac{M_{bal}}{bQ_{bal}}} = \sqrt{\frac{52\times10^6}{1.21\times230}}$ = 432.3 mm

Steel area = $A_{stbal} = \frac{M}{\sigma_{st}jd} = \frac{52\times10^6}{140\times0.87\times432.3}$ = 988 mm^2

Provide 5 nos. of 16 mm diameter bars giving area of 5×201 = 1005 mm^2

Transformed area method:

- A transformed section is a section in which the steel area is replaced by an equivalent concrete area.
- A transformed section consists of a single material, therefore, theory of simple bending can be applied.
- The transformed section may be of steel when concrete is replaced by steel or it may be of concrete when steel area is replaced by concrete. It is usual to replace steel area by concrete, hence a transformed section would mean to a homogenous concrete section.
- The actual concrete in tension zone is absent because we have assumed that concrete carry tensile force. Thus all tensile forces will be carried by steel.



At the centroid of steel reinforcement, the surrounding concrete being elastic and having perfect bond with steel

Strain in steel = strain in concrete

Let f_{st} and f_{cb}' be the stresses in steel and concrete respectively at the level of centroid of steel.

Strain in concrete = strain in steel

$\frac{f_{cb}}{E_c} = \frac{f_{st}}{E_s}$ = $f_{st} = mf_{cb}$'

Now force in steel = $A_{st}f_{st} = A_{st}\ mf_{cb}$'--------------(1)

If this steel is to be replaced by an equivalent concrete area, the equivalent concrete will carry the same force

Now the force in equivalent concrete = transformed area x f_{cb}' ------------ (2)

Equating (1) & (2)

Transformed area x $f_{cb}' = A_{st} m f_{cb}'$

Transformed area = mA_{st}

To find neutral axis:

$b \times x \times (x/2) = mA_{st}(d-x)$

lever arm = $d - (x/3)$

stress in steel $f_{st} = \frac{M}{(d-\frac{x}{3})A_{st}}$

Stress in concrete:

From strain diagram, $\frac{f_{cb}/Ec}{f_{st}/Es} = \frac{x}{d-x}$

$f_{cb} = \frac{f_{st}}{Es/Ec} \times \frac{x}{d-x} = \frac{f_{st}}{m} \times \frac{x}{d-x}$

M.R in compression = $\sigma_{cbc}(bx/2)(L.A)$

M.R in tension = $\sigma_{st}A_{st}(LA)$

Question 4:

A rectangular beam of width 200 mm and effective depth 460 mm reinforced with 3-16 mm dia bar. The section is subjected to a characteristic moment of 30 KNm. Determine the maximum stress in steel and concrete. The materials are M20 grade concrete and mild steel reinforcement. Also find out the M.R of the section.

Solution :

For M20 grade concrete and Fe250

m = 13.33

transformed area of steel = 13.33 × 603

= 8038 mm^2

To find neutral axis, taking moments of transformed area about N.A

$200 \times x \times (x/2) = 8038(460 - x)$

$100x^2 + 8038x - 36987480 = 0$

x = 156.2 mm

lever arm = 460 – (156.2/3) = 407.9 mm

steel stress = $\frac{30\times10^6}{603\times407.9}$ = 121.97 N/mm^2

concrete stress = $\frac{f_{st}}{m} \times \frac{}{d-x} = = \frac{121.97}{13.33} \times \frac{156.2}{303.8}$ = 4.7 N/mm^2

M.R in compression = $\frac{7\times200\times156.2}{2} \times 407.9 \times 10^{-6} = 44.6$ KNm

M.R in tension = $140\times603\times407.9\times10^{-6} = 34.43$ KNm

M.R of the section = 34.43 KNm

Analysis of the section:

Type 1: to find out the depth of neutral axis for a given section and specifying the type of beam

- If the section and steel area are given, find out neutral axis by taking moment of transformed areas about Neutral Axis.

  $b \times x \times (x/2) = mA_{st}(d-x)$

- Find out depth of neutral axis for a balanced section.

  $x = kd$ where = $k = \frac{1}{1+\frac{\sigma st}{m\sigma cbc}}$

- If $x_{act} < x_{bal}$, , - the beam section is under reinforced.

  If $x_{act} > x_{bal}$, , - the beam section is over reinforced

  If $x_{act} = x_{bal}$, , - the beam section is balanced

Type 2 : to find the moment of resistance for a given section

- Find the position of actual and balanced N.A as explained above.
- If $x_{act} < x_{bal}$, the beam section is under reinforced and moment of resistance is given by M.R = $A_{st}\sigma_{st}(d - x/3)$
- If $x_{act} > x_{bal}$, , the beam section is over reinforced and moment of resistance is given by M.R = $b \times x \times (\sigma cbc/2)(d-x/3)$

Design of the section :

Type – 1: dimension not given

- The moment of resistance of balanced section $M_{bal} = Q_{bal}bd^2$. Out of the two variable "b" and "d" , one must known to us. It is usual to fix the width (b) of the section.
- Once width is fixed, the depth will be calculated from following formula

  $d = \sqrt{\frac{M}{Qbal \times b}}$

  here $M = M_{bal}$, where M is the applied moment.

- Then area of steel (A_{st}) will be calculated as per the following formula

  $A_{st} = \frac{M}{\sigma st \times jd}$

Type 2: dimensions are given

- Applied moment M, the section dimensions b and d are given
- Determine $M_{bal} = Q_{bal}bd^2$
- If $M < M_{bal}$, the section is designed as under reinforced beam.
- If $M > M_{bal}$, the section is designed as over reinforced beam.
- If $M = M_{bal}$, the section is designed as balanced.

Question 5:

An R.C.C beam, 300 mm wide and 460 mm effective depth is reinforced with 4 nos. of 12 mm dia bars in tension. Find out the depth of neutral axis and type of the beam. The materials are M20 grade concrete and Fe415 grade steel.

Solution :

For M20 grade concrete, m = 13.33

A_{st} = 4×113 = 452 mm^2

Let x be the depth of N.A

Taking moment of transformed areas about N.A

b×x×(x/2) = mA_{st}(d - x)

(300/2)x^2 = 13.33×452(460 - x)

x^2+40.17x – 18477 = 0

x = 117.3 mm

depth of balanced N.A = 0.29×460 = 133.4 mm

$x_{actual} < x_{bal}$

The beam is under reinforced.

Question 6:

Find the M.R of a beam having width 230 mm and 560 mm effective depth reinforced with 3 nos. of 20 mm dia bar. Also state the type of the beam. The materials are M20 grade concrete and Fe415 grad steel.

Solution:

For M20 grade concrete, m = 13.33

A_{st} = 3×314 = 942 mm^2

Let x be the depth of N.A

Taking moment of transformed areas about N.A

b×x×(x/2) = mA_{st}(d - x)

(230/2)x^2 = 13.33×942(560 - x)

x^2+109.2 x – 61146 = 0

x = 198.6 mm

depth of balanced N.A =kd = 0.29×560 = 162.4 mm

$x_{actual} > x_{bal}$

The beam is over reinforced and concrete will fail first.

M.R = b×x×(σcbc/2)(d-x/3)

= 230 × 198.6 × (7/2)(560 – 198.6/3)× 10^{-6}

= 78.95 Knm

Question 7:

Design a reinforced concrete beam subjected to a bending moment of 20 Knm. Use M20 Grade concrete and Fe 415 grade steel. Keep the width of the beam equal to half the effective depth.

Solution :

For M20 grade concrete, m = 13.33

Neutral axis constant = $k = \frac{1}{1+\frac{\sigma st}{m\sigma cbc}} = \frac{1}{1+\frac{230}{13.33\times 7}}$

= 0.29

Lever arm constant = j = $(1 - \frac{k}{3}) = (1 - \frac{0.29}{3})$

= 0.90

$M_{bal} = Q_{bal}bd^2$

$= ½\sigma_{cbc}kj\ bd^2 = ½×7×0.29×0.90×(d/2)×d$

$= 0.457d^3$

M = 20 Knm = $20×10^6$ Nmm

$M_{bal} = M$

$0.457\ d^3 = 20×10^6$

d = 353 mm

b = 353/2 = 177 mm

steel area = $A_{st} = A_{stbal} = \frac{M}{\sigma_{st} j d}$

$A_{st} = \frac{20×10^6}{230×0.90×353} = 273\ mm^2$

PHILOSOPHY OF LIMIT STATE METHOD

SAFETY AND SERVICEABILITY REQUIREMENTS

In the method of design based on limit state concept, the structure shall be designed to withstand safely all loads liable to act on it throughout its life; it shall also satisfy the serviceability requirements, such as limitations on deflection and cracking. The acceptable limit for the safety and serviceability requirements before failure occurs is called a ‘limit state’. The aim of design is to achieve acceptable probabilities that the structure will not become unfit for the use for which it is intended that it will not reach a limit state.

All relevant limit states shall be considered in design to ensure an adequate degree of safety and serviceability. In general, the structure shall be designed on the basis of the most critical limit state and shall be checked for other limit states.

For ensuring the above objective, the design should be based on characteristic values for material strengths and applied loads, which take into account the variations in the material strengths and in the loads to be supported. The characteristic values should be based on statistical data if available; where such data are not available they should be based on experience. The ‘design values’ are derived from the characteristic values through the use of partial safety factors, one for material strengths and the other for loads. In the absence of special considerations these factors should have the values given in 36 according to the material, the type of loading and the limit state being considered.

Limit State of Collapse

The limit state of collapse of the structure or part of the structure could be assessed from rupture of one or more critical sections and from buckling due to elastic or plastic instability (including the effects of sway where appropriate) or overturning. The resistance to bending, shear, torsion and axial loads at every section shall not be less than the appropriate value at that section produced by the probable most un favourable combination of loads on the structure using the appropriate partial safety factors.

Limit State Design

For ensuring the design objectives, the design should be based on characteristic values for material strengths and applied loads (actions), which take into account the probability of variations in the material strengths and in the loads to be supported. The characteristic values should be based on statistical data, if available. Where such data is not available, they should be based on experience. The design values are derived from the characteristic values through the use of partial safety factors, both for material strengths and for loads. In the absence of special considerations, these factors should have the values given in this section according to the material, the type of load and the limit state being considered. The reliability of design is ensured by requiring that

$$\text{Design Action} \leq \text{Design Strength.}$$

Limit states are the states beyond which the structure no longer satisfies the performance requirements specified. The limit states are classified as

a) Limit state of strength

b) Limit state of serviceability

a) The limit state of strength are those associated with failures (or imminent failure), under the action of probable and most unfavorable combination of loads on the structure using the appropriate partial safety factors, which may endanger the safety of life and property. The limit state of strength includes:

- Loss of equilibrium of the structure as a whole or any of its parts or components.
- Loss of stability of the structure (including the effect of sway where appropriate and overturning) or any of its parts including supports and foundations.
- Failure by excessive deformation, rupture of the structure or any of its parts or components.
- Fracture due to fatigue.
- Brittle fracture.

b) The limit state of serviceability include

- Deformation and deflections, which may adversely affect the appearance or, effective, use of the structure or may cause improper functioning of equipment or services or may cause damages to finishes and non-structural members.
- Vibrations in the structure or any of its components causing discomfort to people, damages to the structure, its contents or which may limit its functional effectiveness. Special consideration shall be given to floor vibration systems susceptible to vibration, such as large open floor areas free of partitions to ensure that such vibrations is acceptable for the intended use and occupancy.
- Repairable damage due to fatigue.
- Corrosion and durability.

Limit States of Serviceability

To satisfy the limit state of serviceability the deflection and cracking in the structure shall not be excessive. This limit state corresponds to deflection and cracking.

Deflection

The deflection of a structure or part shall not adversely affect the appearance or efficiency of the structure or finishes or partitions.

Cracking

Cracking of concrete should not adversely affect the appearance or durability of the structure; the acceptable limits of cracking would vary with the type of structure and environment. The actual width of cracks will vary between the wide limits and predictions of absolute maximum width are not possible. The surface width of cracks should not exceed 0.3mm.

In members where cracking in the tensile zone is harmful either because they are exposed to the effects of the weather or continuously exposed to moisture or in contact soil or ground water, an upper limit of 0.2 mm is suggested for the maximum width of cracks. For particularly aggressive environment, such as the 'severe' category, the assessed surface width of cracks should not in general, exceed 0.1 mm.

CHARACTERISTIC AND DESIGN VALUES AND PARTIAL SAFETY FACTORS

1. Characteristic Strength of Materials

Characteristic strength means that value of the strength of the material below which not more than 5 percent of the test results are expected to fall and is denoted by f. The characteristic strength of concrete (f_{ck}) is as per the mix of concrete. The characteristic strength of steel (f_y) is the minimum stress or 0.2 percent of proof stress.

2. Characteristic Loads

Characteristic load means that value of load which has a 95 percent probability of not being exceeded during the life of the structure. Since data are not available to express loads in statistical terms, for the purpose of this standard, dead loads given in IS 875 (Part l), imposed loads given in IS 875 (Part 2), wind loads given in IS 875 (Part 3), snow load as given in IS 875 (Part **4)** and seismic forces given in IS 1893-2002(part-I) shall be assumed as the characteristic loads.

Design values:

Material:

The design strength of material f_d is given by

$$f_d = \frac{\ }{\gamma_m}$$

where f = characteristic strength of materials
γ_m = partial safety factor appropriate to the material

Loads:

The design load F_d is given by

$F_d = F\gamma_f$

Where F = characteristic load

γ_f = partial safety factor appropriate to the nature of loading

Partial Safety Factors:

1. Partial Safety Factor γ_f for Loads

Sr. No.	Load Combination	Ultimate Limit State	Serviceability Limit State
1	DL + LL	1.5 (DL + LL)	DL + LL
2	DL + WL i) DL contribute to stability ii) DL assists overturning	0.9 DL + 1.5 WL 1.5 (DL + WL)	DL + WL DL + WL
3	DL + LL + WL	1.2 (DL + LL + WL)	DL + 0.8 LL + 0.8 WL

2. Partial Safety Factor γ_m for Material Strength

Sr. No.	Material	Ultimate Limit State	Serviceability Limit State
1	Concrete	1.50	$E_c = 5000\sqrt{f_{ck}}$ MPa
2	Steel	1.15	$E_s = 2 \times 10^5$ MPa

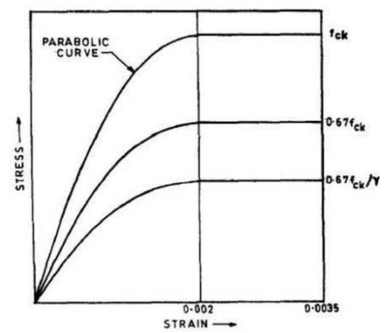
When assessing the strength of a structure or structural member for the limit state of collapse, the values of partial safety factor, should be taken as 1.5 for concrete and 1.15 for steel.

<u>ANALYSIS AND DESIGN OF SINGLE AND DOUBLE REINFORCED SECTION</u>

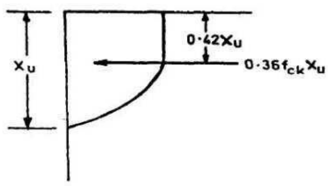
LIMIT STATE OF COLLAPSE: FLEXURE

Assumptions for Limit State of Collapse (Flexure):

1) Plane section normal to the axis remains plane even after bending. i.e. strain at any point on the cross section is directly proportional to the distance from the N.A.
2) Maximum strain in concrete at the outer most compression fibre is taken as 0.0035 in bending.
3) The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test. An acceptable stress strain curve is shown below



STRESS-STRAIN CURVE FOR CONCRETE



STRESS BLOCK PARAMETERS

For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor $\gamma_m = 1.5$ shall be applied in addition to this.

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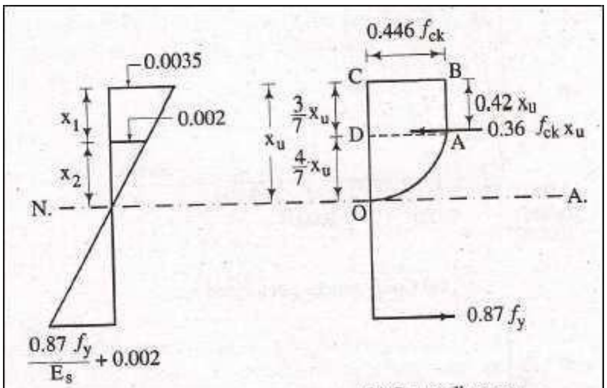


(a) Strain diagram (b) Stress diagram

NOTE –

For the above stress-strain curve the design stress block parameters are as follows:

Area of parabolic portion = (2/3)×0.446f_{ck}×(4/7)x_u

= 0.17$f_{ck}x_u$

Area of rectangular portion = 0.446f_{ck}(3/7)x_u

= 0.19 $f_{ck}x_u$

Total area of stress block = 0.17$f_{ck}x_u$+ 0.19 $f_{ck}x_u$ = 0.36 $f_{ck}x_u$

Let ȳ be the distance of centroid of streee block from the extreme compression fibre

$$\bar{y} = \frac{0.17 fckxu(x_1+\frac{3}{8}x_2)+0.19\ fckxu(\frac{x_1}{2})}{0.36\ fckxu}$$

substituting x_1 = (3/7)x_u

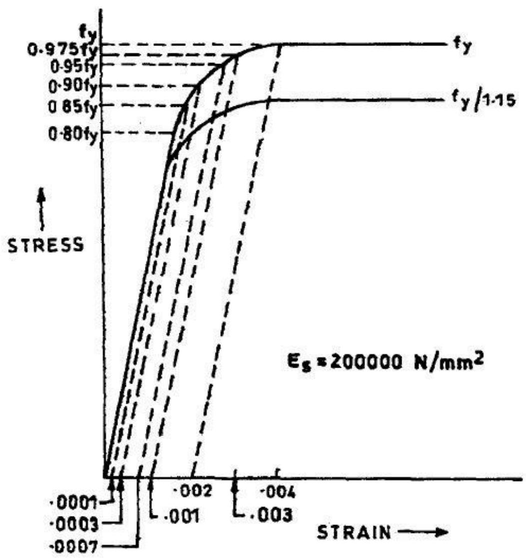
x_2 = (4/7)x_u

Depth of centre of compressive force = ȳ = 0.42x_u from the extreme fibre in compression

Where f_{ck} = characteristic compressive strength of concrete, and x_u = depth of neutral axis

4) the tensile strength of the concrete is ignored.

5) the stresses in the reinforcement are derived from representative stress – strain curve for the type of steel used.



Cold Worked Deformed Bar

6) the maximum strain in tension reinforcement in the section at failure shall not be less than $= \frac{0.87 f_y}{E_s} + 0.002$

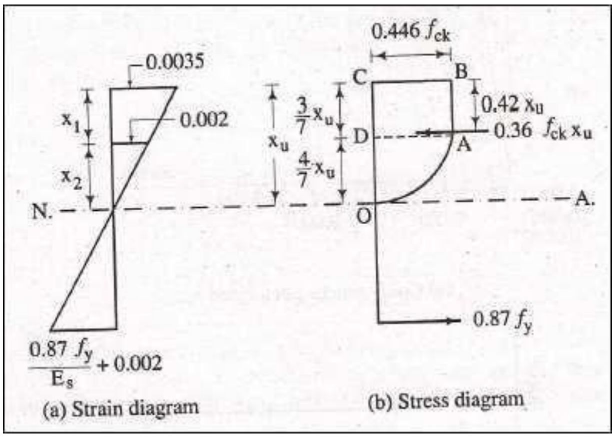
TYPES OF BEAM SECTIONS

- Section in which, tension steel reaches yield strain simultaneously as the concrete reaches the failure strain in bending are called, **'Balanced Section'.**
- Section in which, tension steel reaches yield strain at loads lower than the load at which concrete

  reaches the failure strain in bending are called, **'Under Reinforced Section'**
- Section in which, tension steel reaches yield strain at loads higher than the load at which concrete reaches the failure strain in bending are called, **'Over Reinforced Section'.**

Analysis of single reinforced rectangular beam :

Derivation of formula:



A singly reinforced rectangular beam section with strain diagram and stress diagram are shown in figure.

To find Neutral axis:

Total compression = total tension

$0.36 f_{ck} b x_u = 0.87\ f_y A_{st}$

$X_u = \frac{0.87\ f_y A_{st}}{0.36 f_{ck} b}$

- If $x_u < x_{umax}$, the section is under reinforced section

- If $x_u > x_{umax}$, the section is over reinforced section, then $x_u = x_{umax}$
- If $x_u = x_{umax}$, the section is balanced section

Grade of steel	x_{umax}/d
Fe250	0.53
Fe415	0.48
Fe500	0.46

To find lever arm:

From the stress diagram, the leer arm

$Z = d - 0.42\, x_u$

To find moment of resistance:

(1) For a balanced section

M.R = total compression × lever arm
= total tension × lever arm

$M_u = 0.36\, f_{ck} b\, x_u (d - 0.42\, x_u)$

For limiting value substitute x_{umax} for x_u and M_{ulim} for M_u

$M_{ulim} = 0.36\, (x_{umax}/d)(1 - 0.42\, x_{umax}/d) f_{ck} b d^2$
$= Q_{lim}\, bd^2$

Q_{lim} = limiting moment of resistance factor for balanced rectangular section.

For under reinforced section
$M_u = 0.87 f_y A_{st} (d - 0.42\, x_u)$
$= 0.87 f_y A_{st}\, d\left(1 - \frac{f_y A_{st}}{b d f_{ck}}\right)$

Type of problem:

Three different types of problems are considered for singly reinforced rectangular beams

Type 1:

To find out the depth of neutral axis and specifying the type of the beam

- For a given section, equate total tension and total compression and thus find out the depth of neutral axis using

$$X_u = \frac{0.87\, f_y A_{st}}{0.36 f_{ck} b}$$

- Also find out the limiting value of depth of neutral axis x_{umax}, using $\mathbf{x_{umax}/d}$
- Then if

If $x_u < x_{umax}$, the section is under reinforced section

If $x_u > x_{umax}$, the section is over reinforced section, then $x_u = x_{umax}$

If $x_u = x_{umax}$, the section is balanced section

Type 2:

To find out moment of resistance of a given section

- Find out depth of neutral axis and type of the beam as discussed in type 1
- For over reinforced and balanced section, obtain moment of resistance by using the following equation

$$M_{ulim} = 0.36\,(x_{umax}/d)(1 - 0.42\,x_{umax}/d)fckbd^2$$

- For under reinforced section obtain moment of resistance by using the following equation

$$M_u = 0.36\,f_{ck}b\,x_u(d - 0.42\,x_u)$$

Or

$$M_u = 0.87f_yA_{st}(d - 0.42\,x_u)$$

Type 3:

To design a singly reinforced rectangular section for given width and applied factored moment

The width is usually decided by the functional or architectural requirement.

$$d = \sqrt{\frac{M}{Qlim \times b}}$$

The steel area can be obtained by using the following formula

$$Ast = \frac{Mu}{0.87fy(d - 0.42\,x_{umax})}$$

Question 1:

A rectangular beam 230 mm wide and 520 mm effective depth is reinforced with 4 nos. of 16 mm diameter bars. Find out the depth of neutral axis and specify the type of beam. The materials are M20 grade concrete and Fe415 grade steel. Also find out the depth of neutral axis if the reinforcement is increased to 4 nos. of 20 mm dia bars.

Solution:

Case 1:

$A_{st} = 4\times201 = 804\ mm^2$

Total compression = $0.36fckbx_u = 0.36\times20\times230x_u = 1656\ x_u$

Total tension = $0.87\ f_yA_{st} = 0.87\times415\times804 = 290284$

Equating total compression = total tension

$1656\ x_u = 290284$

$x_u = 175.3$ mm

limiting value of neutral axis = x_{umax}= 048d = 0.48×520 = 250 mm

here $x_u < x_{umax}$, the section is under reinforced section

Case 2 :

$x_u = 175.3$ mm

$A_{st} = 4×314= 1256\ mm^2$

Total compression = $0.36fckbx_u = 0.36×20×230x_u = 1656\ x_u$

Total tension = $0.87\ f_yA_{st}$ = 0.87 ×415×1256 = 453479

$1656\ x_u = 453479$

$x_u = 273.8$ mm

here $x_u > x_{umax}$, the section is over reinforced section

$x_u = x_{umax} = 250$ mm

Question 2:

A rectangular beam 230 mm wide and 460 mm effective depth is reinforced with 3 nos. of 20 mm diameter bars. Find out the factored moment of resistance of the beam. The materials are M20 grade concrete and Fe415 grade steel. Also find out the factored moment of resistance if the reinforcement is increased to 5 nos. of 20 mm dia bars.

Solution:

$A_{st} = 3×314= 942\ mm^2$

Total compression = $0.36fckbx_u = 0.36×20×230x_u = 1656\ x_u$

Total tension = $0.87\ f_yA_{st}$ = 0.87 ×415×942 = 340109

Equating total compression = total tension

$1656\ x_u = 340109$

$x_u = 205.4$ mm

limiting value of neutral axis = x_{umax}= 048d = 0.48×460 = 220.8 mm

here $x_u < x_{umax}$, the section is under reinforced section

$M_u = 0.36\ f_{ck}b\ x_u(d- 0.42\ x_u)$

$= 0.36 ×20×230× 205.4(460- 0.42 ×205.4) = 127.12 × 10^6 = 127$ Knm

Case 2 :

$A_{st} = 4×314= 1256\ mm^2$

Total compression = $0.36 f_{ck} b x_u = 0.36\times20\times230 x_u = 1656\ x_u$

Total tension = $0.87\ f_y A_{st} = 0.87\times415\times1256 = 453479$

$1656\ x_u = 453479$

$x_u = 273.8$ mm

$x_{umax} = 048d = 0.48\times460 = 220.8$ mm

here $x_u > x_{umax}$, the section is over reinforced section

$x_u = x_{umax} = 220.8$ mm

$M_u = 0.36\ f_{ck} b\ x_u(d - 0.42\ x_u)$

$= 0.36\times20\times230\times 220.8(460 - 0.42\times220.8) = 134.2\times10^6 = 134$ Knm

Question 3:

Design a singly reinforced rectangular beam for an applied factored moment of 120 Knm. Assume the width of section as 230 mm. The materials are M20 grade concrete and Fe415 grade steel

Solution:

Mu = 120 Knm

b = 230 mm

fck = 20 N/mm^2

fy = 415 N/mm^2

$$d = \sqrt{\frac{M}{Qlim \times b}} = \sqrt{\frac{120\times10^6}{2.76\times230}} = 434.8 \text{ mm}$$

adopt D = 500 mm

d = 500 – 30 – (20/2) = 460 mm

$$Ast = \frac{Mu}{0.87fy(d - 0.42\ x_{umax})} = \frac{120\times10^6}{0.87\times415(460 - 0.42\times0.48\times460)}$$

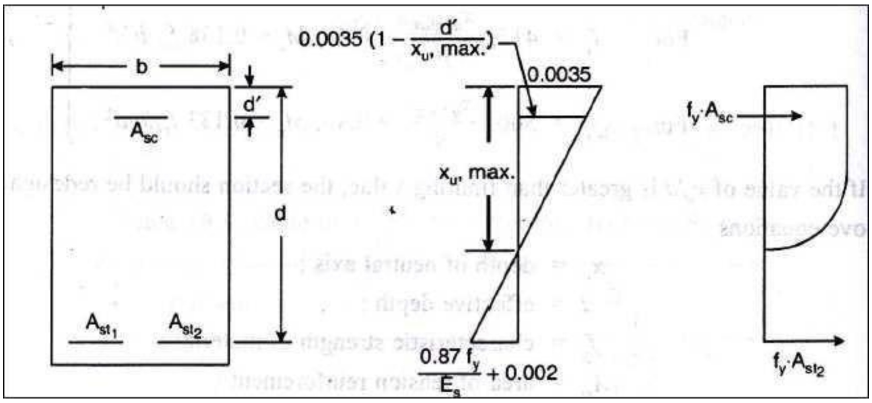
Ast = 904.97 mm^2

Doubly reinforced beam:

If the applied moment is greater than the M.R of a singly reinforced section, there can be three Alternatives

- If possible increase the dimension of the section preferably depth
- Higher grade concrete can be used to increase the M.R of the section
- Steel reinforcement may be added in compression zone to increase the M.R of the section. This is known as doubly reinforced section.

Derivation of the formula



- A doubly reinforced beam section, strain diagram and stress diagram are shown in fig.
- A doubly reinforced beam subjected to a moment Mu can be expressed as a rectangular section with tension reinforcement A_{stlim} reinforced for balanced condition giving moment of resistance M_{ulim}+ an auxiliary section reinforced with compression reinforcement A_{sc} and tensile reinforcement A_{st2} giving moment of resistance M_{u2} such that

$$M_u = M_{ulim} + M_{u2}$$

- For the moment Mulim the tension steel A_{stlim} is found out as explained for singly reinforced beams.
- For the additional moment M_{u2}, the additional tension steel and compression steel are provided such that they give a couple of moment M_{u2}
- Let the compression reinforcement be provided at a depth d' from the extreme compression fibre. Then lever arm for additional moment will be $(d - d')$
- Considering tension steel

$$M_{u2} = 0.87\, f_y . A_{st2}(d - d')$$

Considering compression steel

$$M_{u2} = A_{sc}.(f_{sc} - f_{cc})(d - d')$$

Where, A_{st2} = Area of additional tensile reinforcement

A_{sc} = Area of compression reinforcement

f_{sc} = Stress in compression reinforcement

f_{cc} = Compressive stress in concrete at the level of compression reinforcement

Since the additional reinforcement is balanced by the additional compressive force.

$$A_{sc}.(f_{sc} - f_{cc}) = 0.87\ f_y.A_{st2}$$

$$A_{st2} = \frac{A_s(f_{sc} - f_{cc})}{0.87 f_y}$$

Total area of reinforcement shall be obtained by

$$A_{st} = A_{st1} + A_{st2}$$

A_{st1} = Area of reinforcement for a singly reinforced section for $M_{u,lim}$

The value of f_{sc} in N/mm^2 can be obtained from the following table for different values of d/d' and grade of steel

fy in N/mm^2	d/d'			
	0.05	0.1	0.15	0.20
250	217	217	217	217
415	355	353	342	329
500	424	412	395	370
550	458	441	419	380

Note : The value of f_{cc} is very small as compared to the value of f_{sc} and can be neglected.

Type of problem :

Type 1: to find out the moment of resistance of a given section

- Total compression = total tension

  $C_1 + C_2 = T$

  $0.36 f_{ck} b x_u + A_{sc}.(f_{sc}) = 0.87\ f_y A_{st}$

  Find out x_u

- Find x_{umax} and type of beam
- If $x_u > x_{umax}$, the section is over reinforced section, then $x_u = x_{umax}$
- $M_u = 0.36\ f_{ck} b\ x_u(d - 0.42\ x_u) + A_{sc}.(f_{sc})(d - d')$

Type 2: to find out reinforcement for flexure for a given section and factored moment

- Find out M_{ulim} and reinforcement A_{stlim} for a given section by using the following equation

  $M_{ulim} = 0.36\ f_{ck} b\ x_{umax}(d - 0.42\ x_{umax})$

  $$A_{stlim} = \frac{M_{ulim}}{0.87 f_y (d - 0.42\ x_{umax})}$$

- Obtain $M_{u2} = M_u - M_{ulim}$
- Find compression steel from following equation

$M_{u2} = A_{sc}.(f_{sc})(d – d')$

$A_{sc} = \frac{M_{u2}}{f_s(d-d')}$

- Corresponding tension steel can be found out by from

$A_{st2} = \frac{A_s(f_{sc}-f_{cc})}{0.87f_y}$

- $A_{st} = A_{stlim} + A_{st2}$

Question 1:

Find the factored moment of resistance of a beam section 230 mm wide and 460 mm effective depth reinforced with 2 nos. of 16 mm dia bars as compression reinforcement at an effective cover of 40 mm and 4 nos. of 20 mm dia bars as tension reinforcement. The materials are M20 grade concrete and Fe250 grade steel.

Solution:

b = 230 mm

d = 460 mm

fck = 20 N/mm^2

fy = 250 N/mm^2

d' = 40 mm

$A_{sc} = 2×201 = 402\ mm^2$

$A_{st} = 4×314 = 1256\ mm^2$

$C_1+C_2 = T$

$0.36 f_{ck} b x_u + A_{sc}.(f_{sc}) = 0.87\ f_y A_{st}$

For d'/d = 40/460 = 0.08, next higher value 0.1 may be adopted

$f_{sc} = 217\ N/mm^2$

$0.36×20×230x_u + 402×217 = 0.87 × 250 × 1256$

$1656x_u = 273180 – 87234 = 185946$

$X_u = 112.29$ mm

$X_{umax} = 0.53d = 0.53 × 460 = 243.8$ mm

$x_u < x_{umax}$, the section is under reinforced section

$x_u = 112.29$ mm

$M_u = 0.36\ f_{ck} b\ x_u(d- 0.42\ x_u) + A_{sc}.(f_{sc})(d – d')$

$= 0.36 ×20×230× 112.29(460 - 0.42×112.29) + 402×217(460 – 40)$

= 76.76 + 36.64 = 113.4 Knm

Question 2:

A rectangular beam of size 230 mm wide and 500 mm depth is subjected to a factored moment of 200 Knm. Find the reinforcement for flexure. The materials are M20 grade concrete and Fe415 grade steel.

Solution:

M_u = 200 Knm

$M_{ulim} = 0.36\, f_{ck} b\, x_{umax}(d - 0.42\, x_{umax})$

$= 0.36 \times 20 \times 230 \times 0.48 \times 500(500 - 0.42 \times 0.48 \times 500) = 158.7$ Knm

d’ = 50 mm

d’/d = 50/500 = 0.1

fsc = 353 N/mm^2

$M_{u2} = M_u - M_{ulim} = 200 - 158.7 = 41.3$ Knm

$$A_{sc} = \frac{M_{u2}}{f_{sc}(d-d')} = \frac{41.3\times10^6}{353(500-50)} = 260 \text{ mm}^2$$

$$A_{st2} = \frac{A_s(f_{sc}-f_{cc})}{0.87 f_y} = \frac{260\times353}{0.87\times415} = 254.2 \text{ mm}^2$$

$$A_{stlim} = \frac{\text{Mulim}}{0.87\text{fy}(\text{d} - 0.42\, x_{umax})} = \frac{158.7\times10^6}{0.87\times415(500 - 0.42\times0.48\times500)} = 1101 \text{ mm}^2$$

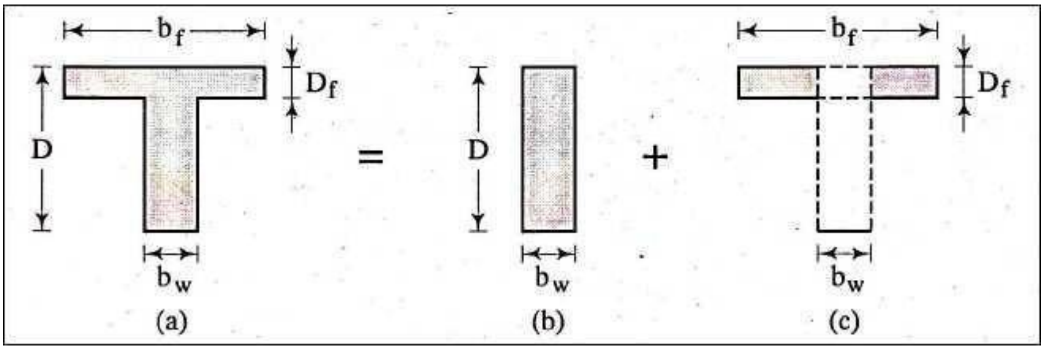
A_{st} = 1101+254 = 1355 mm^2

ANALYSIS AND DESIGN OF T - BEAM

Introduction:

- When there is a reinforced concrete slab over a reinforced concrete beam, the slab and beam can be designed and constructed in such a way that they can act together.
- The concrete in the slab, which is on the compression side of the beam (in the middle portion of continuous beam) can be made to resist the compression force and the tension can be carried by the steel in the tension side of the beam. These combined beam and slab units are called "flanged beam".
- They may be T or L beams depending on whether the slab is on both or only on one side of the beam.
- T or L beams act as flanged beams only between the supports where the bending moment are positive and the slabs are on the compression side of the beam.
- Over the support where the bending moment are negative the slabs are on the tension side then the beam acts only as a rectangular beam with tension steel placed in the slab portion of the beam. Thus at places of negative moment these beams have to be designed as singly or doubly reinforced rectangular beam.

A 'T' beam or 'L' beam can be considered as a rectangular beam with dimensions b_w. D plus a flange of size (b_f - b_w) X D_f . It is shown in the figure beam (a) is equivalent to beam (b) + beam (c).



The flanged beam analysis and design are analogous to doubly reinforced rectangular beam. In doubly reinforced beams additional compressive is provided by adding reinforcement in compression zone, whereas in flanged beams, this is provided by the slab concrete, where the spanning of the slab is perpendicular to that of beam and slab is in compression zone.

If the spanning of the slab is parallel to that of the beam, some portion of slab can be made to span in the direction perpendicular to that of the beam by adding some reinforcement in the slab.

A flanged beam can be also doubly reinforced.

The moment of resistance of a T beam is sum of the moment of resistance of beam (a) is the sum moment of resistance of beam (b) and moment of resistance of beam (c). similarly the steel area required for beam (a) shall be equal to the sum of the steel required for the beam (b) and the steel area required for beam (c).

Position of neutral axis:

For a flanged beam, the Neutral Axis either (a) lies in flange or (b) lies in web

(a) Neutral axis lies in flange (x_u<D_f)

- When the neutral axis lies in the flange, the size of the compression zone becomes $b_f \times x_u$
- As concrete does not resist any tension, the width of tension zone has no effect on the M.R of the section.
- Therefore this beam can be considered as a rectangular beam of dimension $b_f \times d$ and the formula derived for rectangular beams shall be applied.
- For a singly reinforced rectangular beam:
  Equating total compression and total tension
  $X_u = \frac{0.87\ fyAst}{0.36fckb}$
- To find out the type of the beam, x_{umax} shall be found out and compared with actual value of neutral axis x_u

If $x_u < x_{umax}$, the section is under reinforced section

If $x_u > x_{umax}$, the section is over reinforced section, then $x_u = x_{umax}$

If $x_u = x_{umax}$, the section is balanced section

- For under reinforced section

$M_u = 0.36\ f_{ck} b_f\ x_u(d - 0.42\ x_u)$

Or

$M_u = 0.87 f_y A_{st}(d - 0.42\ x_u)$

- For over reinforced section

$M_{ulim} = 0.36\ f_{ck} b_f\ x_{umx}\ (d - 0.42\ x_{umax})$

Problem 1:

A tee beam of effective flange width 1200 mm, thickness of slab 100 mm, width of rib 300 mm and effective depth of 560 mm is reinforced with 4 nos. of 25 mm dia bars. Calculate the factored moment of resistance. The materials are M_{20} grade co concrete and Fe415 grade steel.

Solution :

b_f = 1200 mm

D_f = 100 mm

b_w = 300 mm

d = 560 mm

A_{st} = 1964 mm^2

Assume that N.A lies in flange ($x_u < D_f$)

Total compression = 0.36 $f_{ck} b_f x_u$

Total tension = $0.87 f_y A_{st}$

Equating, total compression = total tension

$0.36 f_{ck} b_f x_u = 0.87 f_y A_{st}$

$0.36 \times 20 \times 1200 x_u = 0.87 \times 415 \times 1964$

x_u = 82.07 mm < D_f

hence N.A lies in the flange.

$x_{umax} = 0.48 \times 560 = 268.8$ mm

$x_u < x_{umax}$, the section is under reinforced section

$M_u = 0.36 f_{ck} b_f x_u (d - 0.42 x_u)$

$= 0.36 \times 20 \times 1200 \times 82.07 (560 - 0.42 \times 82.07) = 372.65 \times 10^6$ Nm

= 372.65 Knm

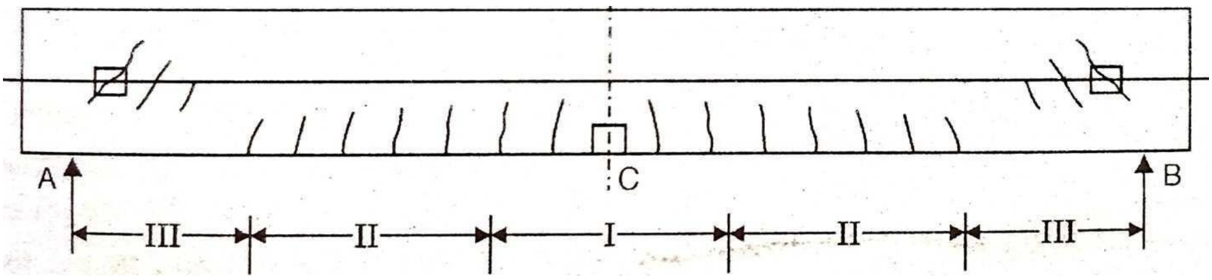
SHEAR STRESS IN REINFORCED CONCRETE BEAMS

When a beam is loaded with transverse loads the Bending Moment(BM) varies from section to section. Shearing stresses in beams are caused by this variation of BM in the beam span. Due to the variation of BM at two sections distance dx apart, there are unequal bending stresses at the same fibre. This inequality of bending stresses produces a tendency in each horizontal fibre to slide over adjacent horizontal fibre causing horizontal shear stress, which is accompanied by complimentary shear stress in vertical direction.

SHEAR CRACKS IN BEAMS:-

Under the transverse loading , at any section of the beam, there exists both Bending Moment(BM) and Shear Force (V).Depending upon the ratio of Bending Moment(BM) to Shear Force(V) at different sections, there may be three regions of shear cracks in the beam as follows.

(a) Region I : Region of flexure Cracks.
(b) Region II : Region of flexure shear Cracks.
(c) Region II : Region of web shear Cracks or diagonal tension cracks.



DIFFERENT REGION OF CRACKS IN BEAMS

(a) Region I : Region of flexure Cracks.

This region normally occurs adjacent to mid-span where BM is large and shear force is either zero or very small.The principal planes are perpendicular to beam axis. When the principal tensile stress reaches the tensile strength of the concrete (which is quite low) tensile cracks develop vertically. The cracks are known as flexural cracks resulting primarily due to flexure.

(b) Region II : Region of flexure shear Cracks. This regions are near the quarter span, to both the sides, where BM is considerable and at the same time Shear force is significant. The cracks in this region are initiated at the tension face, travel vertically (due to flexture) and gradually tend to develop in the inclined direction towards the Nutral Axis(N.A.), as the shear stress goes on increasing towards the N.A. Since the cracks develop under the combined action of BM and Shear, these cracks are known as flexure- shear cracks

(c) Region III : Region of web shear Cracks or diagonal tension cracks.

This regions are adjacent to each support of the beam where S.F is predominant. Since Shear stress is maximum at the N.A., inclined cracks starts developing at the N.A. along the diagonal of an element subject to the action of pure shear.Hence these cracks known as diagonal tension cracks or web-shear cracks.

Design Shear Strength of Reinforced Concrete

Recent laboratory experiments confirmed that reinforced concrete in beams has shear strength even without any shear reinforcement. This shear strength (τ_c) depends on the grade of concrete and the percentage of tension steel in beams. On the other hand, the shear strength of reinforced concrete with the reinforcement is restricted to some maximum value (τ_{cmax}) depending on the grade of concrete. These minimum and maximum shear strengths of reinforced concrete (IS 456, cls. 40.2.1 and 40.2.3, respectively) are given below:

Design shear strength without shear reinforcement (IS 456, cl. 40.2.1)

Table 19 of IS 456 stipulates the design shear strength of concrete τ_c for different grades of concrete with a wide range of percentages of positive tensile steel reinforcement. It is worth mentioning that the reinforced concrete beams must be provided with the minimum shear reinforcement as per cl. 40.3 even when τ_v is less than τ_c given in Table 3

$100A_s/bd$	Concrete grade		
	M20	M25	M30
≤ 0.15	0.28	0.29	0.29
0.25	0.36	0.36	0.37
0.50	0.48	0.49	0.50
0.75	0.56	0.57	0.59
1.00	0.62	0.64	0.66
1.25	0.67	0.70	0.71
1.50	0.72	0.74	0.76

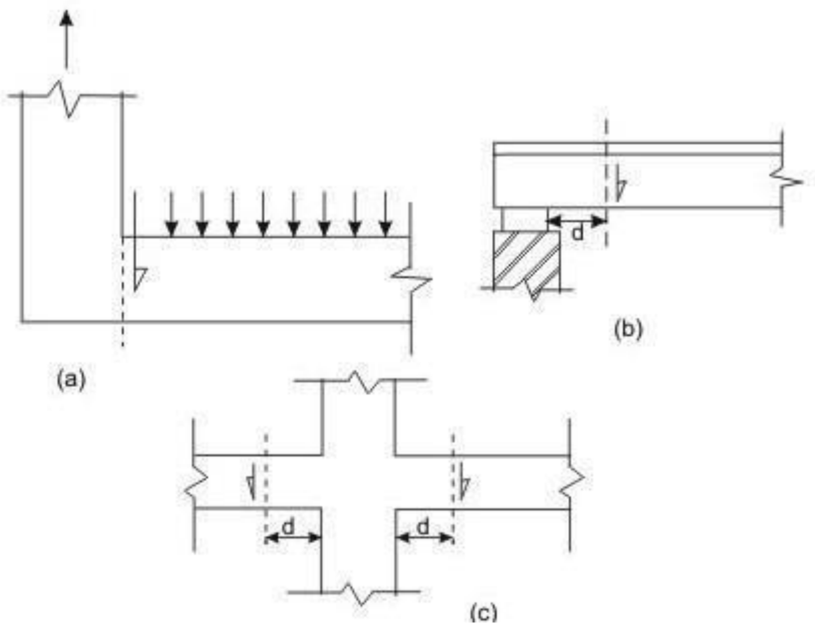
Maximum shear stress with shear reinforcement (cls. 40.2.3, 40.5.1 and 41.3.1)

Table 20 of IS 456 stipulates the maximum shear stress τ_{cmax} of reinforced concrete in beams τ_{cmax} as given below in Table . Under no circumstances, the nominal shear stress in beams τ_v shall exceed τ_{cmax}given in Table for different grades of concrete.

Maximum shear stress τ_{cmax} N/mm^2

Concrete grade	M20	M25	M30
τ_{cmax}	2.8	3.1	3.5

Critical Section for Shear



Clauses 22.6.2 and 22.6.2.1 stipulate the critical section for shear and are as follows:

For beams generally subjected to uniformly distributed loads or where the principal load is located further than 2d from the face of the support, where d is the effective depth of the beam, the critical sections depend on the conditions of supports as shown in Figs. a, b and c and are mentioned below.

(a) When the reaction in the direction of the applied shear introduces tension (Fig.a) into the end region of the member, the shear force is to be computed at the face of the support of the member at that section.

(b) When the reaction in the direction of the applied shear introduces compression into the end region of the member (Figs. b and c), the shear force computed at a distance d from the face of the support is to be used for the design of sections located at a distance less than d from the face of the support. The enhanced shear strength of sections close to supports, however, may be considered as discussed in the following section

Minimum Shear Reinforcement (cls. 40.3, 26.5.1.5 and 26.5.1.6 of IS 456)

Minimum shear reinforcement has to be provided even when τ_v is less than τ_c given in Table 19 as recommended in cl. 40.3 of IS 456. The amount of minimum shear reinforcement, as given in cl. 26.5.1.6, is given below.

The minimum shear reinforcement in the form of stirrups shall be provided such that

$$\frac{A_s}{b s_v} \geq \frac{0.4}{0.87 f_y}$$

Where A_{sv} = total cross-sectional area of stirrup legs effective in shear

S_v = stirrup spacing along the length of the member

b = breadth of the beam or breadth of the web of the flanged beam

f_y = characteristic strength of the stirrup reinforcement in N/mm^2 which shall not be greater than $415N/mm^2$

Further, cl. 26.5.1.5 of IS 456 stipulates that the maximum spacing of shear reinforcement measured along the axis of the nmember shall not be more than 0.75 *d* for vertical stirrups and *d* for inclined stirrups at 45° , where *d* is the effective depth of the section. However, the spacing shall not exceed 300 mm in any case.

Design of Shear Reinforcement (cl. 40.4 of IS 456)

When τ_v is more than τ_c given in Table 19 of IS 456:2000 , shear reinforcement shall be provided in any of the three following forms:

(a) Vertical stirrup
(b) Inclined stirrup
(c) Bent up bars alongwith stirrups

In the case of bent-up bars, it is to be seen that the contribution towards shear resistance of bent-up bars should not be more than fifty per cent of that of the total shear reinforcement.

The amount of shear reinforcement to be provided is determined to carry a shear force V_{us} equal to
$V_{us} = V_u - \tau_c bd$

Where b is the breadth of rectangular beams or b_w in the case of flanged beams

The strengths of shear reinforcement V_{us} for the three types of shear reinforcement are as follows:

(a) **Vertical stirrups** , $V_{us} = 0.87 f_y A_{sv} d \times (1/s_{sv})$

(b) **Inclined stirrup** , $V_{us} = \frac{0.87 f_{yA_{sv}}(sin\alpha + cos\alpha)}{s_v}$

(c) **Bent up bars**, $V_{us} = 0.87 f_y A_{sv} sin\alpha$

Question 1:

A tee beam section having 230 mm width of rib and 460 mm effective depth is reinforced with 5 nos. of 16 mm dia. Bars as tension reinforcement. The section is subjected to a factored shear of 52.5 KN. Check the shear stress and design the shear reinforcement. The materials are M20 grade concrete and Fe415 grade steel. For stirrups mild steel bars may be used.

Solution:

V_u = 52.5 KN

A_{st} = 1005.31 mm^2

Nominal shear stress, $\tau_v = \frac{V_u}{bd} = \frac{52.5 \times 10^3}{230 \times 460}$ = 0.496 N/mm^2

$P_t = \frac{100A_s}{bd} = = \frac{100 \times 1005.31}{bd}$ = 0.95

From table 19 of IS 456, τ_c = 0.608 N/mm^2

Here $\tau_v < \tau_c$, therefore only nominal shear reinforcement is required.

Select 6 mm dia M.S bars for stirrups

$A_{sv} = \frac{2 \times \pi \times 6^2}{4}$ = 56 mm^2 for two legged stirrups

For minimum shear reinforcement = $\frac{Asv}{bs_v} \geq \frac{0.4}{0.87f_y}$

$$\frac{56}{230 \times s_v} \geq \frac{0.4}{0.87 \times 415}$$

$S_v \leq$ 132.4 mm

The spacing shall not exceed

(a) 0.75d = 0.75 × 460 = 345 mm
(b) 300 mm

Provide 6 mm dia two legged stirrup@130 mm c/c

Question 2:

A tee beam section having 230 mm width of rib and 460 mm effective depth is reinforced with 5 nos. of 16 mm dia. Bars as tension reinforcement. The section is subjected to a factored shear of 90 KN. Check the shear stress and design the shear reinforcement. The materials are M20 grade concrete and Fe415 grade steel. For stirrup mild steel bars may be used.

Solution:

V_u = 90 KN

A_{st} = 1005.31 mm^2

Nominal shear stress, $\tau_v = \frac{u}{bd} = \frac{90 \times 10^3}{230 \times 460}$ = 0.85 N/mm^2

$P_t = \frac{100A_s}{bd} = = \frac{100 \times 1005.31}{bd}$ = 0.95

From table 19 of IS 456, τ_c = 0.608 N/mm^2

Here $\tau_v > \tau_c$, therefore shear reinforcement shall be designed.

Shear resistance of concrete = $V_{uc} = \tau_c bd = 0.608 \times 230 \times 460 \times 10^{-3}$ = 64.33 KN

$V_{us} = V_u - \tau_c bd$ = 90 – 64.33 = 25.67 KN

Using 6 mm dia two legged M.S bars for stirrups

$A_{sv} = \frac{2\times\pi\times6^2}{4}$ = 56 mm^2

$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87\times250\times56.55\times460}{25.67\times10^3}$

S_v = 220 mm

The spacing shall not exceed

(a) 0.75d = 0.75 × 460 = 345 mm
(b) 300 mm

Provide 6 mm dia two legged stirrup@130 mm c/c

Question 3:

A tee beam section having 230 mm width of rib and 460 mm effective depth is reinforced with 5 nos. of 16 mm dia. bars as tension reinforcement and out of which 02 nos. bar are bent up at 45°. The section is subjected to a factored shear of 120 KN. Check the shear stress and design the shear reinforcement. The materials are M20 grade concrete and Fe415 grade steel. For stirrups mild steel bars may be used.

Solution:

V_u = 120 KN

Nominal shear stress, $\tau_v = \frac{V_u}{bd} = \frac{120\times10^3}{230\times460}$ = 1.13 N/mm^2

2 bars are bent up

$A_{st} = \frac{2\pi16^2}{4}$ = 402 mm^2

$V_{us} = 0.87 f_y A_{sv} \sin\alpha$ = 0.87 × 415 × 402 × sin 45°

= 102.63 KN

For the remaining 3 bars

$\frac{100A_s}{bd} = \frac{100 \times 3 \times \frac{\pi}{4} \times 16^2}{230 \times 460}$ = 0.57

τ_c = 0.50 N/mm^2

$V_{uc} = \tau_c bd = 0.50 \times 230 \times 460 \times 10^{-3}$ = 52.90 KN

$V_{us} = V_u - \tau_c bd$ = 120 – 52.90 = 67.10 KN

Shear resistance provided by bent up bars = 67.10/2 = 33.55 KN

Shear resistance taken up by stirrups = 67.10 – 33.55 = 33.55 KN

Use 6 mm dia two legged stirrups to resist 33.55 KN

$A_{sv} = \frac{2\times\pi\times6^2}{4} = 56\ mm^2$

$S_v = \frac{0.87 f_y A_{sv} d}{v_{us}} = \frac{0.87\times250\times56.55\times460}{33.55\times10^3} = 168.64\ mm$

The spacing shall not exceed

(a) 0.75d = 0.75 × 460 = 345 mm
(b) 300 mm

Provide 6 mm dia two legged stirrup@130 mm c/c

DEVELOPMENT LENGTH

Introduction:

i) One of the most important assumption in the behavior of reinforced concrete structure is that there is proper bond between concrete and reinforcing bar.
ii) When a RCC element is loaded, the load is first borne by concrete and is then transferred to steel reinforcement.
iii) This transfer from concrete to steel can be effected only when there is no relative movement or slip or sliding between them when any one these two is strained.The force which prevent the slippage between the two constituent material is known as bond.
iv) When steel bar are embedded in concrete, the concrete after setting adheres to the surface of the bar and thus resists any force that tends to pull or push this rod. The intensity of this adhesive force is called bond stress.
v) The bond stresses are the longitudinal shearing stresses acting on the surface between steel and concrete along its length.
vi) Bond stress is also known as the interfacial shear. Hence bond stress is the shear stress acting parallel to the reinforcing bar on the interface between the bar and the concrete.

Types of bond:

1. Flexural bond or local bond
2. Anchorage bond or development bond

Flexural bond:

(i) Flexural bond is one which arises from change in tensile force carried by the bar, along its length, due to change in bending moment along the length of the member. Flexural bond will be more critical at points where shear force is significant.
(ii) Anchorage bond is that which arises over the length of anchorage provided for a bar. It also arises near the endor cutoff point of a reinforcing bar. The anchorage bond resist the pulling out of the bar if t is in tension or pushing in of the bar if it is in compression.

Anchorage bond stress:

Figure shows a steel bar embedded in concrete and is subjected to a tensile force T. due to this force there will be a tendency of the bar to slip out and this tendency is resisted by the bond stress developed over the perimeter of the bar along the length.

The required length necessary to develop full resisting force is called anchorage length in case of axial tension or compression and development length in case of flexural tension and is designated as L_d.

Hence if ϕ is the nominal diameter of the bar we have

$(\pi/4)\phi^2 f_y = \tau_{bd}\pi\phi L_d$

$L_d = \varphi f_y/4\tau_{bd}$

Where f_y is the design stress in steel = $0.87f_y$

$L_d = 0.87f_y\varphi/4\tau_{bd}$

So this indicates that a bar must extend a length L_d beyond any section at which it is required to develop its full strength so that sufficient bond resistance can be developed.

Design bond stress :

(i) The design bond stress in limit state method for plain bars in tension shall be as given in table.

(ii) Design bond stress for deformed bars in tension –
For deformed bars these values shall be increased by 60%

(iii) Design bond stress for bars in compression –
For bars in compression, the values of bond stress for bars in tension shall be increased by 25%.

Standard hooks and bends for end anchorage : Anchorage length:

(i) The development length required at the end of a bar is known as anchorage length.

(ii) Space available at the end of the beam is limited to accommodate the full development length (L_d). In that case hooks or bends are provided. The anchorage value (L_e) of hooks or bend is accounted as contribution to the development length (L_d).

(iii) The minimum radius specified for a hook is 2φ for mild steel bar and 4φ for high yield bar.

(iv) In the case of deformed bars the value of bond stress for various grades of concrete is greater by 60% than the plain bars. Hence the deformed bars may be used without hooks provided anchorage requirements are satisfied.

(v) The length of straight bars beyond the end of the curve should be atleast 4 times the diameter of the bars.

Code requirement for anchoring reinforcing bars:

(i) **Anchoring bars in tension** :

Deformed bars may be used without end anchorages provided development length requirement is satisfied. Hooks should normally be provided for plain bars in tension. The anchorage value of bend shall be taken as 4 times the diameter of bar for each 45 bend subjected to a maximum of 16 times the diameter of the bar. The anchorage value of standard U- type hook shall be equal to 16 times the diameter of the bar.

(ii) Anchoring bars in compression :

The anchorage length of straight bar in compression shall be equal to the development length of bars in compression.

(iii) Anchoring shear reinforcement :

Inclined bars – The development length shall be as for bars in tension : this length shall be measured as under :

(i) In tension zone from the end of the sloping or inclined portion of the bar.

(ii) In compression zone from mid depth of the beam

Stirrups –

When the bar is bent through an angle of at least 90˚ round a bar of atleast it will continued beyond the end of the curve for a length of atleast eight diameter or when the bar is bent through an angle of 135 and is continued beyond the end of the curve for a length of at least six bar diameter or when the bar is bent through an angle of 180 and is continued beyond the end of the curve for a length atleast four bar diameter.

Checking development length of tension bars:

The stress in a reinforcing bar, at every section must be developed on both side of the section. This is done by providing development length L_d to both the sides of the section. Such a development length is usually available at mid span location where positive (or sagging) B.M is maximum for simply supported beams. Similarly such a development length is usually available at the intermediate support of a continuous beam where negative (or hogging) B.M is maximum. Hence no special checking is necessary for such locations. But special checking for development length is essential at following locations;

i) At simple supports.
ii) At Cantilever supports.
iii) At point of contraflexure.
iv) At point of bar cut off.

Requirement of development length:

The code stipulates that at the simple support the positive moment tension reinforcement shall be limited to a diameter such that

$L_d \leq (M_1/V)+L_0$

Where L_d = development length

M_1 = M.R of the section assuming all reinforcement at the section to be stressed to f_{yd}

V = shear force at the section due to design loads

L_0 = sum of anchorage beyond the centre of support and the value of L_0 is limited to d or 12φ whichever is greater

The code further recommends that the value of M_1/V may be increased to 30% when the ends of the reinforcement are confined by a compressive reaction. Such situation arises when a beam is simply supported over a wall.

Thus at simple supports where the compressive reaction confines the ends of reinforcing bar, we have

$L_d \leq 1.3(M_1/V)+L_0$

Question 1:

Calculate the anchorage length in tension and compression for

(a) Single mild steel bar of dia φ in concrete of grade M20
(b) An HYSD bar of grade Fe415 of dia # in concrete of grade M20

Solution :

(a) M.S bar

(i) In tension:

Design stress for M.S bar = $\sigma_s = 0.87f_y = 0.87\times250 = 217.5\ N/mm^2$

$\tau_{bd} = 1.2\ N/mm^2$

anchorage length = development length#

$= 0.87f_y\varphi/4\tau_{bd}$

$= 217.5\varphi/4\times1.2$

$= 45.3\varphi$ or 46φ

(ii) Compression:

$\sigma_s = 0.87f_y = 0.87\times250 = 217.5\ N/mm^2$
$\tau_{bd} = 1.2 \times 1.25$ (for compression) $= 1.5\ N/mm^2$

$L_d = 0.87f_y\varphi/4\tau_{bd} = 217.5\varphi/4\times1.5$

$= 36.3\varphi = 37\varphi$

(b) HYSD bar

(i) Tension :

$\sigma_s = 0.87f_y = 0.87\times415 = 361\ N/mm^2$
$\tau_{bd} = 1.2 \times 1.6$ (HYSD bar) $= 1.92\ N/mm^2$
$L_d = 361\#/4\tau_{bd} = 217.5\varphi/4\times1.92 = 47\#$

(ii) Compression :

$\sigma_s = 0.87f_y = 0.87\times415 = 361\ N/mm^2$
$\tau_{bd} = 1.2 \times 1.6$ (HYSD bar) $\times 1.25$ (comp) $= 2.4\ N/mm^2$
$L_d = 361\#/4\tau_{bd} = 217.5\varphi/4\times2.4 = 37.6\# = 38\#$

Question 2:

A simply supported nbeam is 25 cm by 50 cm and has 2-20 mm HYSD bars going into the support. If the shear force at the centre of support is 110 KN at working loads, determine the anchorage length. Assume M20 mix and Fe415 grade steel.

Solution:

Factored S.F = 1.5×110= 165 KN

$A_{st} = 2\times(\pi/4)d^2 = 628\ mm^2$

Assuming 25 mm clear cover to the longitudinal bar

Effective depth = 500- 25 – (20/2) = 465 mm

$F_y = 415\ N/mm^2$

$M_1 = 0.87 f_y A_{st}(d-0.42x_u)$

$X_u = \frac{0.87\ fyAst}{0.36fckb}$

$= \frac{0.87\times415\times628}{0.36\times20\times250} = 125.97 = 126mm$

$X_{umax} = 0.48d = 0.48\times465 = 223.2\ mm$

$X_u < X_{umax}$

$M_1 = 0.87\times415\times628(465-0.42\times126)$

$= 93.43\times10^6$ Knm

$\tau_{bd} = 1.2\times1.6 = 1.92$

$L_d = 0.87\times415\varphi/4\times1.92$

$= 47\varphi$

If the bar is given 90® bend at the centre of the support its anchorage value $L_0 = 8\varphi = 8\times20 = 160$ mm

$L_d \le 1.3(M_1/V)+L_0$

$47\varphi \le \frac{1.3\times93.45\times10^6}{165\times10^3} + 160$

$\varphi \le 19$ mm

since actual bar diameter of 20 mm is greater than 19 mm there is a need to increase the anchorage

length L_0 to 12φ i.e 240 mm

$L_d \le 1.3(M_1/V)+L_0$

$47\varphi \le \frac{1.3\times93.45\times10^6}{165\times10^3} + 240$

$\varphi \le 20.8$ mm

DESIGN OF BEAMS

Design procedure:

The procedure for design of a beam may be summarized as follows:

(i) Estimation of load:
- The correct estimation of loads, a beam has to bear, leads to an economical and safe design of the beams.
- A designer should not forget to account for any possible load acting on the structure as it leads to an under design of the member and subsequent failure of the beam.
- The dead load on the beam may be self weight of the beam, floor finish, partitions and some special fixed load if specified.
- The live loads shall be different for different structure depending on the functional use of the structure.

(ii) Analysis:
- Using the above determined loads, the shear force and bending moment are found outand diagrams drawn.

(iii) Design :

After analysis design the beam as follows:
- Using the maximum moment, calculate the depth of beam required for balanced section.
- Find out the steel area required for design moment.
- Check the shear stress and development length of bars.
- If some bars are curtailed, check for curtailment using curtailment rules.
- Check the deflection and cracking using rules for control of deflection and cracking.
- Draw complete sketches of designed beam with elevation and section.

Basic rules for design:

(i) Effective span :

Simply supported beam or slab: The effective span of a member that is not built integrally with its support shall be taken as clear span plus the effective depth of slab or beam or centre to centre of support whichever is less.

(ii) Control of deflection:

The deflection of a structure or part there of shall not adversely affect the appearance or efficiency of the structure or finishes or partitions. For beams and slabs the vertical deflection limit may generally be assumed to be satisfied provided that the span to depth ratios are not greater than the values obtained as below:

(a) Basic values of span to effective depth ratios for span upto 10 m:

Cantilever ------------ 7
Simply supported ------------------- 20
Continuous 26

(b) For span above 10 m the value in (a) may be multiplied by a factor 10/span in meter except in case of cantilever.

(c) Depending on the area and the type of steel for tension reinforcement the vales in (a) & (b) shall be multiplied by modification factor as per graph on pg no.38.

F_s = the steel stress of service loads
= $0.58f_y$ (area of cross section of steel required/area of cross section of steel provided)

(iii) Reinforcement in beams:

(a) Tension reinforcement :

Minimum reinforcement – the minimum area of the tension reinforcement shall not be less than that given by the following:

$$\frac{A_{st}}{bd} = \frac{0.85}{f_y}$$

Maximum reinforcement – the maximum area of tension reinforcement shall not exceed 0.04bD

(b) Compression reinforcement:

Maximum reinforcement – the maximum area of compression reinforcement shall not exceed 0.04bD

(iv) Criteria for development length:

- According to clause 26.2 of IS:456, the calculated tension or compression in any bar at any section shall be developed on each side of the section by an appropriate development length.
- No bar can be bent up or curtailed upto a distance of development length from the point of maximum moment.
- For example for Fe415 grade reinforcement, the development length in concrete of grade M20 in tension is $47\neq$. If 20 mm diameter bar is used, the bars cannot be bent or curtailed upto a distance of $47\times 20 = 940$ mm from the point of maximum bending moment.
- The code stipulates that at the simple support the positive moment tension reinforcement shall be limited to a diameter such that

  $L_d \leq (M_1/V)+L_0$

  Where L_d = development length

  M_1 = M.R of the section assuming all reinforcement at the section to be stressed to f_{yd}

  V = shear force at the section due to design loads

  L_0 = sum of anchorage beyond the centre of support and the value of L_0 is limited to d or 12φ whichever is greater
- The code further recommends that the value of M_1/V may be increased to 30% when the ends of the reinforcement are confined by a compressive reaction. Such situation arises when a beam is simply supported over a wall.

  Thus at simple supports where the compressive reaction confines the ends of reinforcing bar, we have

  $L_d \leq 1.3(M_1/V)+L_0$

(v) Slenderness limit for beams :

To ensure lateral stability of a beam as per clause 23.3 of IS:456

For simply supported and continuous beam

Clear span β 60b

$$\beta\frac{250b^2}{d}$$

For cantilever beams

Clear span β 25b

$$\beta\,\frac{100b^2}{d}$$

Question 1:

A simply supported rectangular beam of 6m span carries a uniformly distributed characteristic load of 24 KN/m inclusive of self weight of 24 KN/m. Design the beam. The materials are grade M20 concrete and HYSD reinforcement of grade Fe415. The beam is resting on RCC column.

Solution:

Factored load = 1.5 × 24 = 36 KN/m

$M_u = \frac{36\times6^2}{8} = 162$ KNm

$V_u = \frac{36\times6}{2} = 108$ KN

(a) Calculation of depth:

Assume width of the section b = 300 mm

$d_{req} = \sqrt{\frac{M_u}{Q_{lim}\times b}}$

$= \sqrt{\frac{162\times10^6}{2.76\times300}} = 442.32$ mm

Provide effective depth d = 500 mm
Assume clear cover of 30 mm and 20 mm dia Fe415 bar
Overall depth = D = 500+30+10 = 540 mm

(b) Calculation of steel area:

$Ast = \frac{Mu}{0.87fy(d - 0.42\,x_{umax})} = \frac{162\times10^6}{0.87\times415(500 - 0.42\times0.48\times500)}$

$= 1123.97\ mm^2$

Provide 4 nos. of 20 mm dia bar giving Ast = 1256.64 mm^2

Let 2 bars are bent at 1.25D = 1.25 × 540 = 675 mm from the face of the support.

The remaining bars should extend within the support for a distance of $L_d/3 = \frac{47\times20}{3} = 313$ mm

(c) Check for development length:

(i) A bar can be bent up at a distance greater than L_d = 47≠ from the centre of the support i.e 47 × 20 = 940 mm
In this case the distance is = 3000 – 940 = 2060 mm (safe)

(ii) For the remaining bars
$A_{st} = 628\ mm^2$

$M_{u1} = 0.87 f_y A_{st} d(1-\frac{A_{st}f_y}{bdf_{ck}})$

$= 0.87\times 415 \times 628 \times 500 (1- \frac{628\times415}{300\times500\times20})$

= 103.62 KNm

V_u = 108 KN

L_0 = 12≠ (assumed)

As the reinforcement is confined by compressive reaction

$L_d \leq 1.3(M_1/V)+L_0$

$\frac{\emptyset\sigma_s}{4c_{bd}} \leq 1.3\frac{103.62\times10^6}{108\times10^3}+12\times20$

$47\phi \leq 1.3\frac{103.62\times10^6}{108\times10^3}+12\times20$

Φ ≤ 31.64 mm

Φ provided = 20 mm (safe)

The remaining bars should extend within the support for a distance of $L_d/3 = \frac{47\times20}{3}$ = 313 mm. If support Width is 300 mm , the bars extend for 150 + L_0 = 150 + (12×20) = 390 mm within the support.

(d) Check for shear:

V_u = 108 KN

$\tau_v = \frac{V_u}{bd} = \frac{108\times10^3}{300\times500}$ = 0.72 N/mm^2

$100A_s/bd = \frac{100\times628}{300\times500}$ =0.42

τ_c = 0.44 N/mm^2 < τ_v

hence shear design is necessary.

At support , $V_{us} = V_u - V_{uc}$

= $(108\times10^3) - (0.44\times300\times500)$ = 42000 N

= 42 KN

Capacity of bent up bars to resist shear = $0.87f_yA_{sv}\sin\alpha$

= 0.87×415×628×sin45˚ = 160.33 KN

Bent up bars share 50% = 21 KN

Vertical stirrup share 50% = 21 KN

At distance d, where bent up bars are not available

V_{us} = 42 KN

Design stirrup for shear = 42 KN

Using 6mm dia mild steel stirrup

A_{sv} = 56 mm^2

$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}$

$42 \times 10^3 = \frac{0.87 \times 250 \times 56 \times 500}{s_v}$

$S_v = 145$ mm

Spacing should not exceed

(i) 0.75d = 0.75 × 500 = 375 mm
(ii) 300 mm
(iii) 145 mm

Provide 6mm dia two legged mild steel stirrup @ 140 mm c/c.

(e) Check for deflection:

Basic span/d ratio = 20

Service stress = $0.58 f_y \frac{A_{streg}}{A_{stpro}}$

$= 0.58 \times 415 \times \frac{1123.97}{1256.64}$

$= 215.29$ N/mm^2

$100A_s/bd = \frac{100 \times 1256.64}{300 \times 500} = 0.84$

Modification factor = 1.15

Span/d permissible = 20×1.15 = 23

Actual span/d = 6000/500 = 12 (safe)

ANALYSIS AND DESIGN OF SIMPLY SUPPORTED SLAB

Slab - Slabs are plate elements having the depth D much smaller than its span and width. They usually carry a uniformly distributed road and form the floor or roof of the building.

They are generally of two types :-

(1) **One way spanning slab**-

(i) The slab supported on all four edges but $l_y/lx > 2$. Here as ly is much more than lx then there will be a tendency of the slab to bend in one direction (about lx) only. Hence the slab where $l_y/lx > 2$ is called one way slab provided that it is supported on all four edges.

(ii) The slab supported on two opposite support is a one way spanning slab.

(2) **Two way panning slab**-

If the slab is supported on all four edges and if $l_y/l_x \leq 2$, the tendency of the slab is to bend in both direction. Such slabs are called two way spanning slab.
The following conditions should be satisfied

(i) The slab shall be supported on all four edges.

(ii) $ly/lx \leq 2$.

Analysis of one way slab :-

The analysis of the slab spanning in one direction is done by assuming it to be a beam of 1m width.

In addition to the main reinforcement, transverse reinforcement (also called distribution reinforcement) is also provided in a direction at right angles to the main reinforcement. The transverse reinforcement is provided to serve the following purpose-

(i) It resists the temperature and shrinkage stresses.

(ii) It keep the main reinforcement in position.

(iii) It distributes the concentrated and non uniforms load throughout the slab more evenly and uniformly.

Basic rules for design :-

(1) Effective span – Thus in case of freely supported slab the effective span is taken equal to the distance between centre to centre of supports or the clear distance between the supports plus the effective depth of the slab whichever is less.

(2) Control of deflection- The basic values of span to effective depth
Cantilever ----7
Simply supported---- 20
Continuous ----26

(3) Reinforcement Requirements:

(i) Minimum reinforcement- The mild steel reinforcement in either direction in slab shall not be less than 0.15% of the total cross sectional area.
For HYSD bar this value is 0.12% of the total cross sectional area.

(ii) Maximum diameter- The diameter of the bar shall not exceed one eighth of the total thickness of the slab.

(iii) Minimum diameter- SP:34 gives the guideline for minimum diameter of bars in slabs. Accordingly, for main bars the minimum diameter shall be 10 mm for plain bars and 8 mm for deformed bars. For distribution bars, the minimum diameter shall be 6 mm in any case.

(4) Shear Stress- In normal cases the shear in slabs is not critical. However shear shall be checked in accordance with the code requirements of clause 40.2. For solid slab the design shear strength in concrete shall be $k\tau_c$.

(5) Deflection- This shall be checked in the same manner as the beams. The slabs are thin elements and deflection may govern the thickness of the slab.

(6) Cracking- To ensure that the cracking of the slabs is not excessive, spacing of the reinforcement shall be limited to the followings:

For main bars, spacing β 3d
β 300 mm
For secondary bars, spacing β 5d
β450 mm
Where d= Effective depth of slab .

(7) Cover: For mild exposure, specified clear cover is 20 mm. This can be reduced by 5 mm when the reinforcement of 12 mm diameter or less is used.

(8) Development Length : The development length in the slab shall be checked in the same manner as for beams.

For checking development length, L_0 may be assumed as 8# for HYSD bars (usually end anchorage is not provided) and 12ϕ for mild steel.

Question 1:

A simply supported one way slab of clear span 3.0 m is supported on masonry walls of thickness 350 mm. slab is used for residential loads. Design the slab. The materials are grade M20 concrete and HYSD reinforcement of grade Fe415. Live load shall be 2 KN/m^2.

Solution:
Assume overall depth of slab D = 130 m

(a) Load calculation:

Dead load = 0.13×25 = 3.25 KN/m^2
Floor finish = 1.00 kN/m^2
Live load = 2.00 KN/m^2

Total load = 6.25 KN/m^2

Factored load = 1.5 × 6.25 = 9.4 KN/m^2
Effective span (1) 3000 + 350 =3350 mm c/c support
3000 +110 (effective depth) = 3110 mm

Use3.11 m effective span.

(b) Calculation of bending moment and shear force:

Consider 1 m length of slab

Load =9.4 kN/m

Maximum moment = 9.4 x 3.11^2 /8 = 11.36 kNm.

Maxium shear = 9.4 x3/2 =14.1 kN (based on clear span)

(c) Calculation of depth:

$$d_{req} = \sqrt{\frac{M_u}{Q_{lim} \times b}}$$

$$= \sqrt{\frac{11.36 \times 10^6}{2.76 \times 1000}} = 64.15 \text{ mm}$$

$d_{provided}$ = 130 – 15 (cover) – 5 (assume 10ϕ bar)
= 110 mm

(d) Calculation of steel area:

$$Ast = \frac{Mu}{0.87fy(d - 0.42\, x_{umax})} = \frac{11.36 \times 10^6}{0.87 \times 415(110 - 0.42 \times 0.48 \times 110)}$$

= 358 mm^2

$$\text{Spacing} = \frac{area\ of\ one\ bar \times 1000}{required\ area\ in\ mm^2 per\ metre}$$

$$= \frac{\frac{\pi \times 8^2}{4} \times 1000}{358} = 140 \text{ mm}$$

Provide 8mm dia bar @ 140 mm c/c giving A_{st} = $\frac{area\ of\ one\ bar \times 1000}{spacing} = \frac{\frac{\pi \times 8^2}{4} \times 1000}{140}$ = 360 mm^2

Half the bars are bent at 0.1l = 0.1 × 3100 = 310 mm

Remaining bars provide 180 mm^2 area.

$$p_t = \frac{100A_s}{bD} = \frac{100 \times 180}{1000 \times 130} = 0.13 > 0.12$$

i.e remaining bars provide minimum steel. Thus half bars may be bent up.

Distribution steel = $\frac{0.15 \times 1000 \times 130}{100}$ = 195 mm^2, using mild steel

$$\text{Spacing} = \frac{area\ of\ one\ bar \times 1000}{required\ area\ in\ mm^2 per\ metre}$$

$$= \frac{\frac{\pi \times 6^2}{4} \times 1000}{195} = 145 \text{ mm}$$

Provide 6mm ϕ @ 140 mm c/c

(e) Check for shear:

For bars at support

Correct d = 130 – 15 – 4
= 111 mm

$p_t = \frac{100A_s}{bd} = \frac{100\times180}{1000\times111} = 0.16$

for slab upto 150 mm thickness, k = 1.3

value of τ_c from IS:456 = 0.28 N/mm^2

design shear strength = $k\tau_c = 1.3\times0.28 = 0.364$N/mm^2

actual shear stress = $\tau_v = \frac{14.1\times10^3}{1000\times111} = 0.13$ N/mm^2 < 0.364 N/mm^2 (safe)

(f) Check for development length:

For continuing bars, A_{st} = 180 mm^2. Also , the ends of the reinforcement are confined by compressive reaction.

$M_{u1} = 0.87f_y A_{st} d(1-\frac{A_{st}f_y}{bdf_{ck}})$

$= 0.87\times 415 \times 180 \times 111\ (1- \frac{180\times415}{1000\times111\times20})$

= 7 KNm

V_u = 14.1 KN

L_0 = 12≠ (assumed)

$L_d \leq 1.3(M_1/V)+L_0$

$\frac{\O\sigma}{4c_{bd}} \leq 1.3\ \frac{7\times10^6}{14.1\times10^3} + 12\times8$

$47\phi \leq 1.3\frac{7\times10^6}{14.1\times10^3} + 6$

$\Phi \leq 15.77$ mm

Φ provided = 8 mm (safe)

(g) Check for deflection:

Basic span/d ratio = 20

Service stress = $0.58f_y\frac{A_{streg}}{A_{stpro}}$

$= 0.58\times415\times\frac{358}{360}$

= 240 N/mm^2

$100A_s/bd = \frac{100\times360}{1000\times111} = 0.32$

Modification factor = 1.5

Span/d permissible = 20×1.5 = 30

Actual span/d = 3100/111 = 28 < 30 (safe)